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Theoretical Analysis of Regional Groundwater Flow.

2. Effect of Water-Table Configuration and Subsurface Permeability Variation

R. ALLAN FREEZE

Water Research Branch, Department of Mines and Technical Surveys
Calgary, Canada

P. A. WITHERSPOON

Department of Civil Engineering, University of California, Berkeley

Abstract. Details of steady-state flow in regional groundwater basins can be investigated using digital computer solutions of appropriately designed mathematical models. The factors that must be considered are: (1) ratio of depth to lateral extent of the basin; (2) water-table configuration; and (3) stratigraphy and resulting subsurface variations in permeability. The results of this study provide a theoretical basis for the following properties of regional flow systems: (1) groundwater discharge will tend to be concentrated in major valleys; (2) recharge areas are invariably larger than discharge areas; (3) in hummocky terrain, numerous sub-basins are superposed on the regional system; (4) buried aquifers tend to concentrate flow toward the principal discharge area, have a limiting effect on sub-basins, and need not outcrop to produce artesian flow conditions; (5) stratigraphic discontinuities can lead to distributions of recharge and discharge areas that are difficult to anticipate and that are largely independent of the water-table configuration. (Key words: Groundwater; computers, digital; drainage basin characteristics)

INTRODUCTION

Regional groundwater flow in a nonhomogeneous, anisotropic system can be investigated by means of mathematical models. In Part 1 of this study [*Freeze and Witherspoon,* 1966], steady-state solutions have been obtained by analytical and numerical methods. The numerical methods, which involve computer solutions to finite-difference equations, proved to be superior to the analytical methods and were recommended for both two-dimensional and three-dimensional models of groundwater basins.

In this study, potential nets obtained from two-dimensional hypothetical models are examined qualitatively to show the effect of variations in the controlling parameters on the regional groundwater flow system. We have chosen only a limited number of examples from a much more comprehensive study [*Freeze,* 1966] to illustrate some important points. Much more of this kind of work can be done, but we believe that these examples will demonstrate the usefulness of the digital computer approach to investigations of regional groundwater flow.

In Part 3 of this study, the quantitative significance of the mathematical model approach will be discussed with field examples.

It is first necessary to define some of the terms that appear throughout this study. Natural groundwater *recharge* refers to water that percolates down through the unsaturated zone to the water-table and actually enters the dynamic groundwater flow system. This definition excludes that portion of the moisture surplus that enters the ground and increases the soil moisture content but does not enter the flow pattern itself. The term is not to be confused with the actual areal precipitation which, in some cases, may lead to groundwater recharge and in other cases may not. Natural groundwater *discharge* is water that is discharged from the dynamic groundwater flow system by means of stream baseflow, springs, seepage areas, and evapotranspiration. We have not considered the effects of artificial discharge (wells and well fields) in this study.

A *discharge area* is an area where the direction of groundwater flow is toward the water-table. A *recharge area* is an area where the direction of groundwater flow is away from the water-table. It is convenient to introduce the concept of a *hinge-line,* which is a line on the surface of the water-table that separates a discharge area from a recharge area. Its projection on a two-dimensional vertical section will be a point on the water-table.

A *groundwater basin* is a three-dimensional closed system that contains the entire flow paths followed by all the water recharging the basin. The flow pattern within a given basin may be simple, involving only one recharge area and one discharge area, or complex, involving many. A two-dimensional section through a groundwater basin is representative of the three-dimensional basin if it is taken parallel to the direction of dip of the water-table slope.

In this study the *water-table* is considered to be an imaginary surface beneath ground level at which the absolute pressure is atmospheric. It is assumed to be the upper boundary of the saturated flow system [*Freeze and Witherspoon,* 1966].

The potential function used throughout this study is the hydraulic head φ, which is defined by

$$\varphi = \phi/g$$

where

ϕ = hydraulic potential [*Hubbert,* 1940]
g = acceleration due to gravity

At any point in the potential field, φ equals the elevation above a standard datum of the liquid level in a piezometer inserted at that point.

The manner in which any flow system is defined by a potential field is governed by the interrelations between three governing factors: (1) shape of the region in which the potential field is defined; (2) existing boundary conditions; and (3) nature of the inhomogeneities in the flow properties of the region.

In our case, the potential field is a region represented by a two-dimensional, vertical cross section through a groundwater basin. The region is roughly rectangular with vertical impermeable sides, a horizontal impermeable base, and an irregular upper boundary (the water-table). There are two ways by which we can control the shape of this region: (1) by changes in the water-table configuration, which result in small but important changes in shape, or (2) by changes in the ratio of depth to lateral extent of the basin. By varying this ratio, we can examine all cases from that of a deep basin of limited lateral extent to the more usual case of a shallow groundwater basin of broad extent.

At the external boundaries, the controlling conditions are explicitly defined by the mathematical model; only the water-table configuration can be altered. Within the region, the property of the medium that affects the nature of the potential field is, of course, the permeability. We must, therefore, examine the effect of inhomogeneity and anisotropy of permeability on groundwater flow patterns.

Tóth [1962, 1963] has used an analytical solution to arrive at results for the homogeneous case that show the effect of varying the 'depth : lateral extent' ratio. For this study, we have therefore chosen a fixed basin size and attempted to isolate the effects of water-table configuration and subsurface permeability variations.

Each of the diagrams in Figures 1 through 6 depicts the potential pattern in a vertical cross section through a hypothetical groundwater basin. In some cases, random flowlines are included to clarify the direction of flow. The diagrams are true scale and dimensionless, i.e., all dimensions are given in terms of s, the total length of the basin. The diagrams could thus represent flow patterns for systems covering only a few acres or for those extending over many hundreds of square miles.

For this discussion, we have chosen a basin with a 'depth : lateral extent' ratio of 1:12. Depth was arbitrarily taken as measured at the shallowest point, i.e., at the left-hand edge of each diagram. The total relief on the water-table surface was arbitrarily set at $\frac{1}{6}$ the basin depth for all water-table configurations. The machine-plotted equipotential lines were all constructed using a constant $\Delta\varphi$, which was $\frac{1}{60}$ of the total available head.

Permeability contrasts are noted on most diagrams. Where a single value is shown within a given formation, the formation is isotropic. In anisotropic cases, both horizontal and vertical permeabilities are indicated. Where no permeability is given, the formation is homogeneous and isotropic.

These permeabilities may also be considered dimensionless, because it is the permeability ratio that controls the nature of the potential field. For example, in Figure 2A the same potential net would result from permeabilities of 10 and 100 as exists for 1 and 10. The quantity of flow through the basin would, of course, be different.

In some diagrams the closeness of the equipotential lines was such that only every other or every third line has been reproduced to avoid a maze of lines. In these instances, the position of the missing lines is indicated by tick marks, and the resulting gaps in the potential fields must not be interpreted as sudden changes in gradient.

EFFECT OF WATER-TABLE CONFIGURATION

The investigation of the effect of the water-table configuration on regional groundwater flow patterns was begun by *Tóth* [1962, 1963]. He considered two cases: a constant gentle regional slope such as one would expect to find in the flat prairie, and a water-table with the configuration of a sine curve, as one might expect in hummocky terrain. With the increased versatility of the methods introduced in Part 1 of this study, we can now investigate water-table configurations of a more irregular nature, and ones more representative of actual field conditions.

In Figure 1, the effect of three different water-table configurations on the flow through a homogeneous isotropic medium is shown. The following general comments can be made:

1. In the recharge areas, the equipotential lines meet the water-table obliquely with the acute angle on the upslope side. In the discharge areas, the acute angle is on the downslope side. At the hinge-line, the equipotential meets the water-table at right angles.

2. The existence of a high in the water-table configuration, whether it be a major regional high or a minor reversal in slope, results in recharge at that point and for some distance

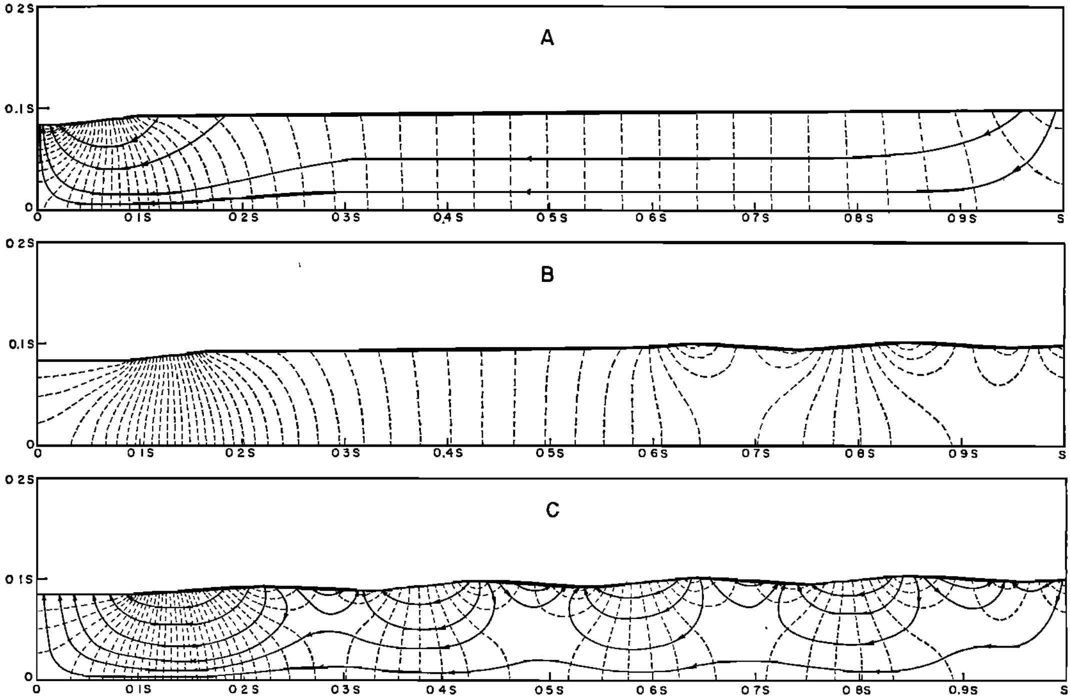

Fig. 1. Effect of water-table configuration on regional groundwater flow through homogeneous isotropic mediums.

on either side. The existence of a low results in discharge at that point and for some distance on either side.

3. If the spacing between equipotential contours is x feet, then an equipotential line must meet the water-table at every point along its length that represents an increase in elevation of x feet. Steep water-table slopes therefore result in many equipotential lines and high gradients near the water-table (and indeed to some depth). Shallow slopes are conducive to low gradients. Where the water-table is flat, it represents an equipotential surface, and vertical gradients result.

4. *Tóth* [1962] showed that a gentle constant regional water-table slope over a homogeneous medium results in flow which is essentially horizontal. Recharge is concentrated at the upstream end of the recharge area; discharge, at the downstream end of the discharge area. The hinge-line is at the midpoint. By contrast, when there is a major valley present (Figure 1A), the hinge-line occurs midway up the steep valley flank. Discharge is concentrated in the valley. Recharge occurs over the entire upland area but is concentrated in two locations: at the upstream end of the recharge area and at the break in slope above the steep valley flank.

5. The existence of a hummocky water-table configuration (Figure 1C) results in numerous sub-basins within the major groundwater basin. Water that enters the flow system in a given recharge area may be discharged in the nearest topographic low or may be transmitted to the regional discharge area in the bottom of the major valley. In essence, Figure 1C shows the effect of the addition of a major valley to *Tóth*'s [1963] sine curve configuration (although the hummocks in Figure 1C are not sine curves but straight line segments). Figure 1B shows a composite situation involving a major valley, an area of constant slope, and a hummocky water-table in the upstream end of the region.

It should be noted that the second comment above holds only for two-dimensional sections taken parallel to the direction of dip of the water-table slope. Section EE′, Figure 9, in Part 1 of this study [*Freeze and Witherspoon*, 1966] provides a counter example of a random two-dimensional section through a three-dimensional basin in which a topographic low (near the intersection of CC′) acts as a recharge area. Section EE′ crosses this valley in its upstream portion, where longitudinal components of flow parallel to the direction of the valley are more important than lateral flow components parallel to the section.

EFFECT OF SUBSURFACE PERMEABILITY VARIATION

Figures 2 through 6 consist of eighteen potential nets designed to display the diversity of regional flow patterns that can arise from the consideration of a variable stratigraphy and its resulting subsurface permeability variations. The principles are best shown using the simplest realistic water-table configuration, that of Figure 1A. Thirteen of the diagrams utilize this configuration, whereas the other five show the effect of stratigraphy on flow in hummocky basins.

Layered Cases

Comparison of Figure 2A with Figure 1A shows the effect of the introduction of a layer with a permeability 10 times that of the overlying beds. The lower formation is, in effect, an aquifer with essentially horizontal flow that is being recharged from above. As a consequence, the vertical component of flow in the upper layer is much more pronounced than it was in the homogeneous case (Figure 1A). One should also note the downstream increase in gradient within the aquifer of Figure 2A, caused by an increasing number of flowlines that enter the aquifer from the upper layer. Discharge is concentrated in the valley bottom; the entire constant regional slope is a recharge area.

These effects can be further altered by considering a higher permeability contrast. Figures 2B and 2C show the effect of increasing the permeability of the basal aquifer. As the permeability ratio increases, the following changes can be noted:

1. The vertical upward or downward flow through the overlying low-permeability layer becomes more pronounced. For example, in the upstream recharge areas at the right-hand side of the diagrams, the flow becomes more vertical, the vertical flow exists over a larger area, and the vertical gradient increases.

2. The horizontal gradient in the aquifer de-

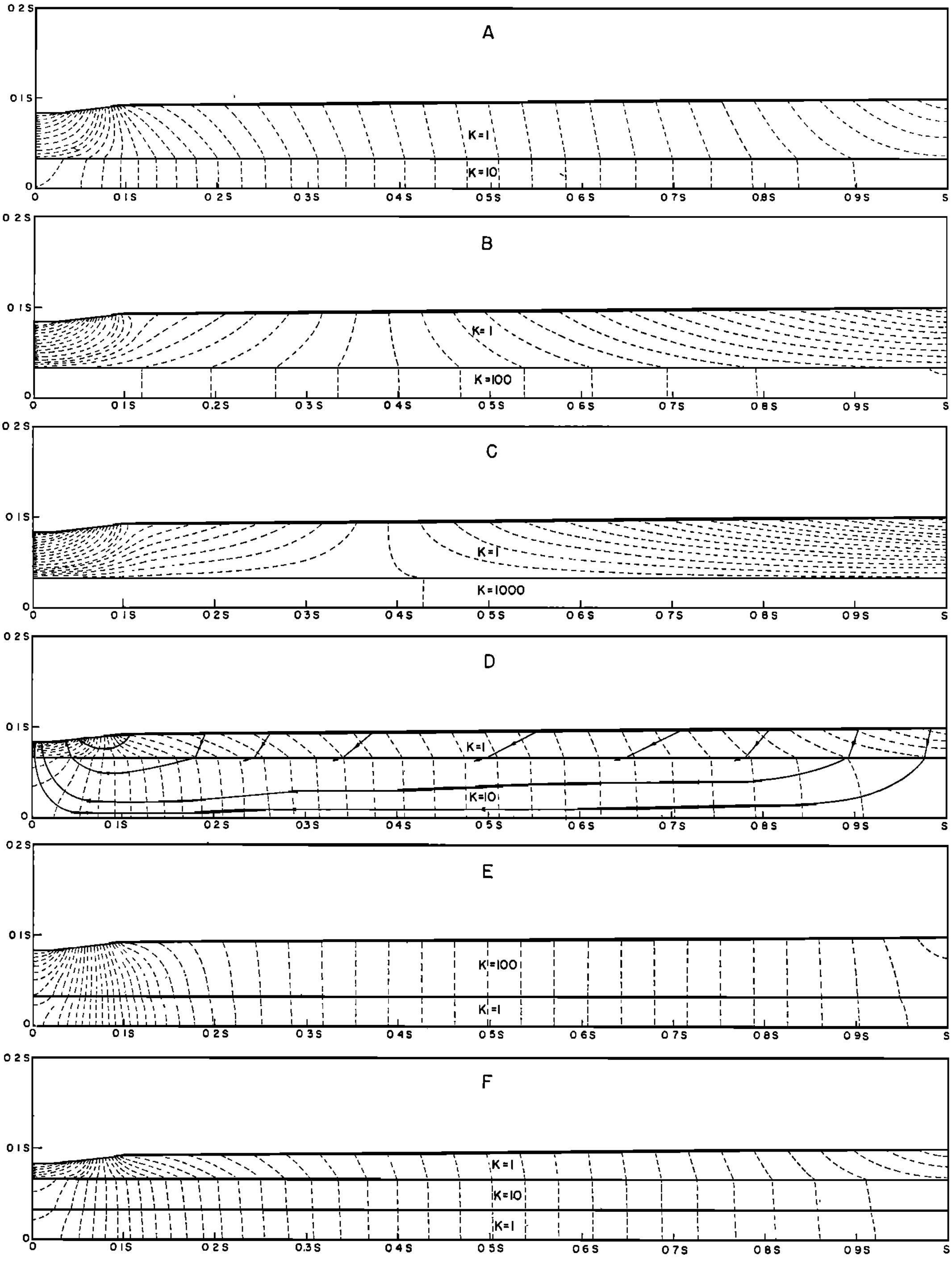

Fig. 2. Regional groundwater flow through layered mediums with a simple water-table configuration.

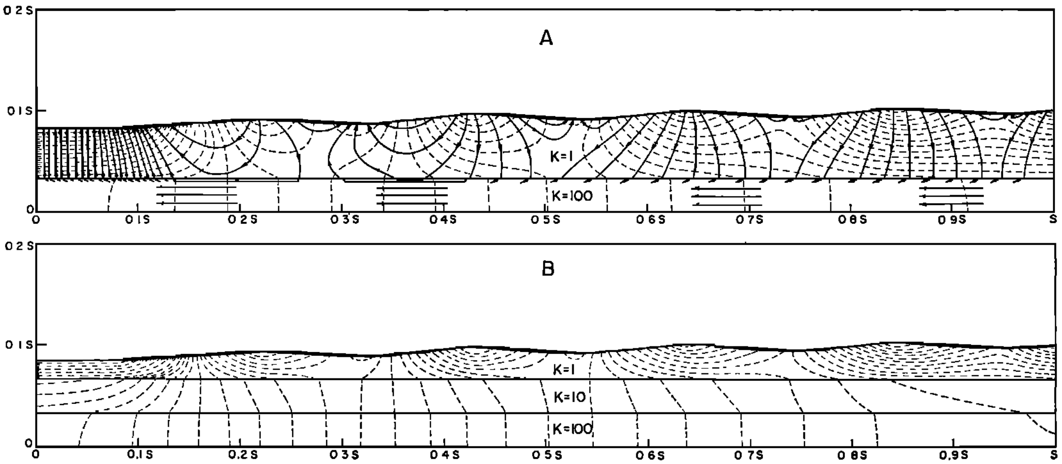

Fig. 3. Regional groundwater flow through layered mediums with a hummocky water-table configuration.

creases, but the quantity of flow (which can be calculated using Darcy's Law) increases.

3. The hinge-line moves upslope, creating larger discharge areas. This is a result of the increased quantity of water flowing through the basal aquifer, which must escape as the influence of the left-hand vertical impermeable boundary is felt. The magnitude of this effect may not be entirely realistic, as it is possible that, for permeability ratios of 1000:1 (Figure 2C), the 'major valley' at the left of the diagram would not create an imaginary impermeable boundary. Horizontal flow through the aquifer might proceed to the left until a more pronounced topographic influence was encountered.

The thickness of the basal aquifer has little effect on the nature of the flow pattern, as shown by a comparison of Figures 2D and 2A. The quantity of water flowing through the system represented by Figure 2A would, of course, be about half that flowing through the system of Figure 2D.

Comparison of Figures 2E and 1A shows that the flow pattern resulting from a two-layer case, when the upper layer has the larger permeability, is almost identical to that of the homogeneous case. The quantity of flow is, of course, considerably different in each case. The fact that a geologic configuration exists that results in a potential pattern identical to that of the homogeneous case emphasizes the fact that good permeability data are necessary before results of piezometer installations can be interpreted successfully in terms of quantitative estimates of regional flow.

Figure 2F shows a three-layer case with the aquifer in the middle of the section. Again we see that a low permeability layer beneath the aquifer has little or no effect on the potential pattern (compare Figures 2E and 2F).

Figure 3 shows the effect of stratigraphic variations on regional groundwater flow when the water-table has a hummocky surface. As shown in Figure 3A, the effect of the basal aquifer is to intensify the downward flow through the upper layer. The aquifer provides a highway for flow that passes under the upper layer and restricts the sub-basins in the hummocky region considerably. Figure 3A should be compared with Figure 1C. In effect, the relative importance of the discharge area in the major valley has been increased many fold owing to the presence of the buried aquifer.

It should be noted that the conditions of artesian flow inferred by the term 'confined aquifer' are met by several of the potential nets presented in Figures 2 and 3. For example, if a piezometer were sunk into the basal aquifer in Figure 2C, the static water level would lie considerably above the top of the aquifer. In the vicinity of the major valley, flowing artesian wells would occur.

Partial Layers and Lenses

The effect of lenticular bodies of high permeability and the particular importance of their position in the basin are shown in Figure 4.

The presence of a partial basal aquifer in the upstream half of the basin (Figure 4A) results in a discharge area that occurs in the middle of the constant regional slope. The occurrence of such a discharge area under strictly topographic control would, of course, be impossible. The majority of the flow that has entered the system in the upper half of the basin is discharged at this point. What was originally a single basin in the homogeneous case has become two basins under the influence of the partial aquifer.

When the partial basal acquifer occurs in the downstream half of the basin (Figure 4B), the central discharge area does not exist and, indeed, recharge in the region over the aquifer is concentrated. The zone of most intensive recharge is thus shifted from the upstream portion of the basin in the homogeneous case to the downstream portion because of the presence of the partial aquifer.

Figure 4C shows the aquifer as a stratigraphic lens in the regional basin. In this case, there is recharge over the upstream end of the lens and discharge over the downstream end. There is horizontal flow through the lens as well as in the low permeability layer beneath. Figure 4D shows the effect of a lens on the potential field in a hummocky basin.

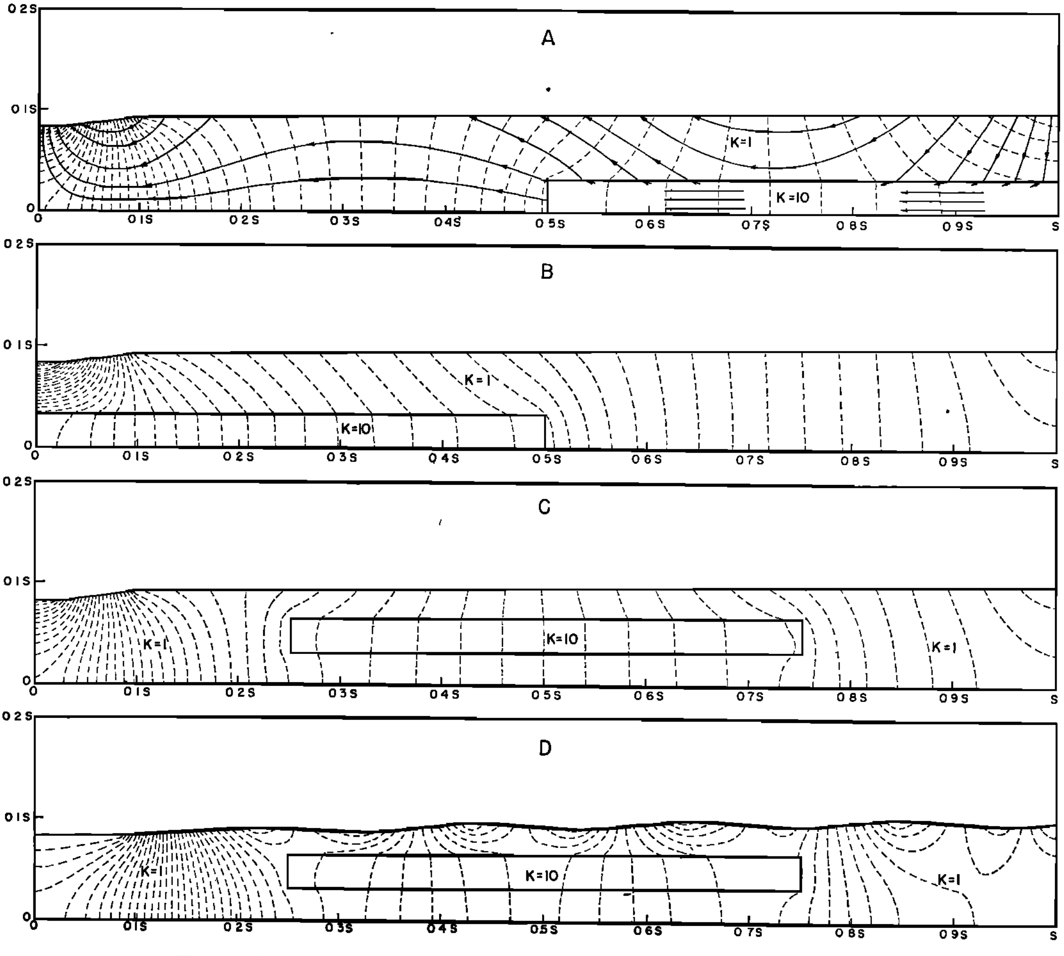

Fig. 4. Regional groundwater flow through partial layers and lenses.

Sloping Stratigraphy

Figure 5 presents three examples of the effect of sloping stratigraphy. In Figure 5A there is an increased recharge where the $K = 10$ layer outcrops, which, of course, is to be expected. The point where the upper boundary of this layer meets the water-table is a major hingeline with a large discharge area downstream. Above the break in slope, a second recharge area is evident, and as usual, the major discharge is concentrated in the valley.

An interesting pair of flowlines to consider is shown in Figure 5B. Here, the difference of a few feet in the point of recharge will make the difference between the water entering a minor sub-basin or the major regional system of groundwater flow. In effect, what was thought to be a single basin when we set up this model is, in reality, two basins. If the sloping bed is reversed as in Figure 5C, a considerably different potential field is developed.

Anisotropic Formations

When anistropy exists, the problem of analyzing regional groundwater flow becomes more complex. The digital computer approach is well suited to the analysis of such problems, and Figure 6 shows three simple cases to illustrate the method.

Figures 6A and 6B show the effect of anisotropy on the regional groundwater flow pattern in a homogeneous medium bounded by the same simple water-table slope of Figure 1A. In Figure 6A the horizontal permeability is 10 times the vertical. For illustrative purposes, this situation is reversed in Figure 6B. The effect of these permeability configurations is best realized by comparing the flowlines of these figures with those of Figure 1A for the isotropic case. Figure 6C is a two-layer case that combines the anisotropy conditions of Figures 6A and 6B in one system. This system could be representative of a vertically fractured formation overlying a horizontally stratified layer.

Considerable care must be exercised in the construction of flowlines in anisotropic mediums, since the flowlines will not in general intersect the equipotentials at right angles. Two methods are available. The first [*Maasland*, 1957] utilizes the transformed section, whereby

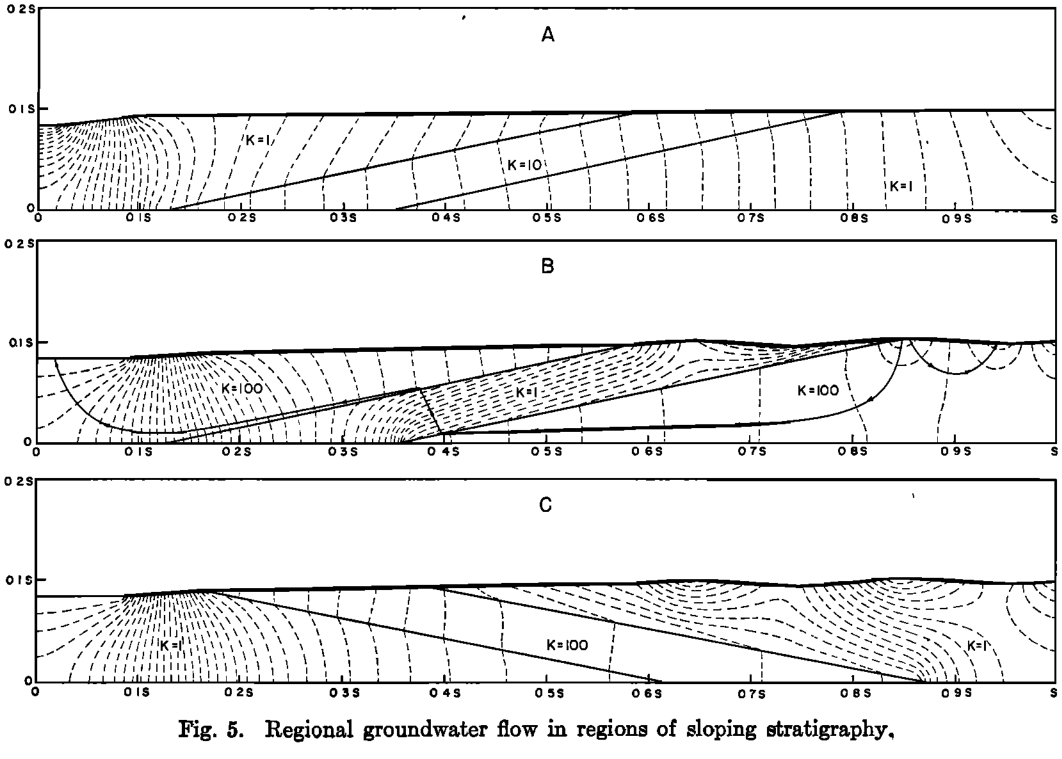

Fig. 5. Regional groundwater flow in regions of sloping stratigraphy.

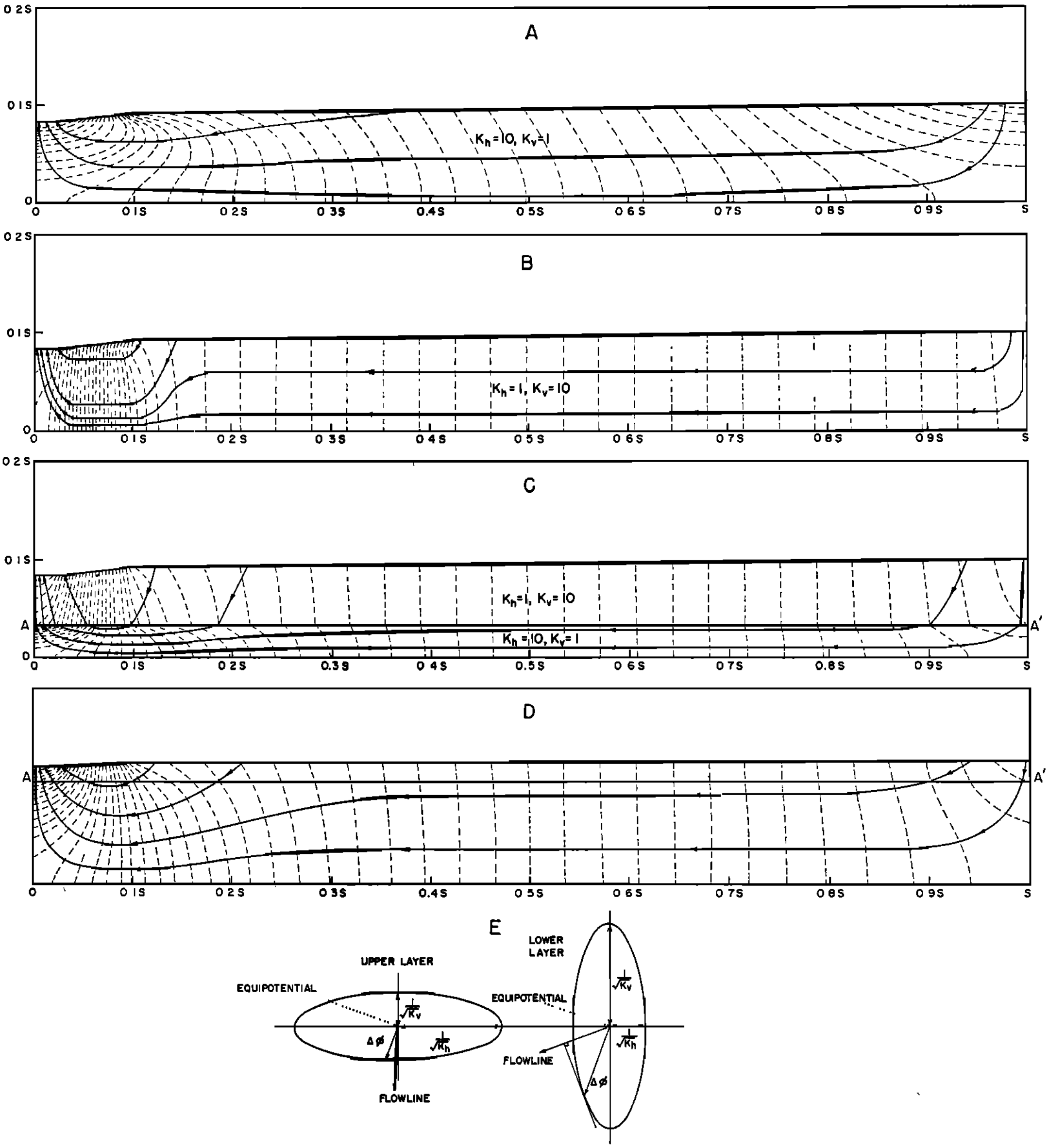

Fig. 6. Effect of anisotropy on regional groundwater flow.

an equivalent homogeneous isotropic system is obtained by suitably expanding or shrinking the coordinates of each point in the anisotropic medium. The transformation is

$$x' = (K_0/K_h)^{1/2}x$$

$$y' = (K_0/K_v)^{1/2}y$$

where x and y are the original coordinates; x' and y' the transformed coordinates; K_h and K_v the horizontal and vertical permeabilities; and K_0 is an arbitrary constant having the dimensions of K_h and K_v.

Figure 6D shows the transformed section for the two-layer case of Figure 6C. In the upper layer, we choose $K_0 = 1$; then $x' = x$, $y' = y/\sqrt{10}$, and the vertical dimension is reduced by a factor of $\sqrt{10}$. In the lower layer, we choose $K_0 = 10$; then $x' = x$, $y' = \sqrt{10}\,y$, and the vertical dimension is expanded by a factor of $\sqrt{10}$. The position of the interlayer boundary AA′ is shown on both diagrams.

Once the equipotentials have been transferred from the real (Figure 6C) to the transformed (Figure 6D) section, a homogeneous isotropic flownet can be drawn in the transformed section and the flowlines transferred back to the true case. Several random flowlines are shown to illustrate the direction of flow.

The method of the transformed section indicates that the effect of varying the anisotropic ratio in a homogeneous medium is identical to that of varying the 'depth : lateral extent' ratio. The diagrams presented by *Tóth* [1963], which were designed to show the effect of the 'depth : lateral extent' ratio in a homogeneous basin bounded by a hummocky water-table, could therefore also be interpreted in terms of the effects of anisotropy.

A second method has recently been described [*Liakopoulos*, 1965] whereby the direction of flow at any point in an anisotropic medium can be determined with the aid of the permeability ellipse and without the necessity of a transformed section. Figure 6E shows the permeability ellipses for both upper and lower layers of Figure 6C. The direction of flow at any point can be obtained graphically as follows:

1. Draw a vector in the direction of the hydraulic gradient (i.e., perpendicular to the equipotential at the point in question).
2. Draw a tangent to the ellipse at the point where the vector cuts the ellipse.
3. The direction of flow is perpendicular to the tangent line.

In the constructions shown on the two ellipses in Figure 6E, the direction of the hydraulic gradient is the same in each case. The resulting direction of flow, however, is radically different and is dependent on the prevailing direction of anisotropy.

It should be noted that all the cases treated in Figure 6 have axes of anisotropy that coincide with the coordinate directions. The more general case of a skewed anisotropy requires a more complicated mathematical approach utilizing the concept of permeability in tensor form. Numerical solutions employing the finite element method [*Zienkiewicz et al.*, 1966] appear to be well suited to this problem.

DISCUSSION OF RESULTS

The water-table configuration and the variations in subsurface permeability have been identified throughout this study as the broad governing factors that control regional groundwater flow within a basin of given size. Both these properties of the basin can exist in an infinite variety, but certain generalizations are evident on the basis of the potential patterns that have been developed in this study.

Distribution of Discharge and Recharge Areas

Areas of groundwater discharge occur under the influence of at least six distinguishable cases of water-table configuration and geologic setting.

1. The existence of a valley of sufficient magnitude to create an imaginary vertical impermeable boundary that extends to the full depth of the basin will cause concentrated groundwater discharge into the valley.
2. Minor topographic lows will also cause discharge areas. The sub-basins may be sufficient to capture the flow from the entire depth of the basin [*Tóth,* 1963] or may be restricted to the upper layers in a stratified system (Figure 3A).
3. A break in slope even though both slopes are positive may be sufficient to cause small quantities of groundwater discharge just below the steep components of slope. This phenomenon is illustrated in Figure 1B at the base of the first hummock near the middle of the diagram.
4. Discharge areas that are entirely the result of geologic control can occur at the surface above the pinchout of a buried high-permeability aquifer. The extent of the discharge area and the intensity of discharge depend on: (a) the position of the partial aquifer within the basin; and (b) the permeability contrast between the aquifer and the surrounding medium.
5. A discharge area can be created below the outcrop of a downstream sloping aquifer (Figure 5A).
6. A discharge area can occur at the outcrop of an upstream sloping aquifer (Figure 5C).

As can be seen in the various illustrations, areas of recharge are widespread. However, concentrations of recharge can be expected in the following situations.

1. Near the topographic divide at the upstream end of a homogeneous isotropic basin with a simple water-table configuration (Figure 1A).
2. In the upstream half of basins with continuous layered stratigraphy (Figures 2B and 2C).
3. On and just above steep valley slopes (Figure 1).
4. On water-table highs in regions with a hummocky water-table configuration (Figures 1C, 3A, and 3B).
5. In the area overlying the upstream portion of a partial aquifer (Figure 4).
6. In the outcrop area of a downstream sloping aquifer (Figure 5A).

It is clear that the distribution of recharge and discharge areas is affected by the presence of anisotropy. From the limited number of examples that we have examined, few generalizations can be made. However, two observations are worthy of note: (1) it appears that where the horizontal permeability is significantly greater than the vertical, the quantities of groundwater entering the system are more evenly distributed over the recharge area; and (2) Figure 6B brings out the paradoxical fact that high vertical permeabilities do not necessarily lead to large rates of vertical recharge.

In general, discharge areas are smaller than recharge areas. In the two-dimensional hypothetical models of this study, the discharge area occupies between 7% and 40% of the total length of the basin. Results of studies with three-dimensional models show that the percentage in actual groundwater basins is near the lower end of this range.

Depth and Lateral Extent of Groundwater Basins

It has been shown that hummocky water-table configurations are conducive to the establishment of small sub-basins within the major basin. Under these circumstances, the concept of a total basin yield is negated, and each component basin must be considered separately. It is logical, therefore, to examine the factors that control the depth and lateral extent of these sub-basins.

In the homogeneous case (Figure 1C) the majority of flow takes place near the surface in small sub-basins, but a certain amount of flow by-passes these near surface systems to enter the major flow system. At least one flow path traverses the entire basin. *Tóth* [1963] has shown that the influence of the hummocks increases as: (1) the amplitude of the hummocks increases, and (2) the 'depth : lateral extent' ratio decreases. The groundwater basin may even be broken up into a series of small sub-basins with no flow traversing the entire basin.

The effect of introducing a high-permeability aquifer into the system is to create a highway for groundwater flow such that the percentage flow traversing the entire basin increases. The percentage of total flow that enters the basin-wide flow system depends on three parameters:

1. The permeability ratio between the aquifer and other formations;
2. The 'depth : lateral extent' ratio of the basin;
3. The percentage of the total depth taken up by the high-permeability layer.

There is very little generalization possible regarding the effect of irregular geologic configurations. It is safe to say, however, that the introduction of discontinuities (partial layers, lenses, and sloping beds) will result in the formation of small sub-basins that did not exist in the homogeneous case.

Effective Depth to a Basal Impermeable Boundary

One of the basic assumptions of this study [*Freeze and Witherspoon,* 1966] requires the presence of a horizontal impermeable boundary at some depth. Another assumption suggests that there is no such thing as a completely impermeable formation. The resolution of this seeming paradox lies in the fact that there are certain geologic configurations that have the same effect on the potential pattern as an impermeable boundary.

Figure 2A shows a two-layer case with a simple water-table configuration. The equipotential lines cross the $K = 10$ layer vertically and meet the assumed basal impermeable boundary perpendicularly, as they must. In Figure 2F, another low permeability layer has been added beneath the aquifer. The effect on the flow pattern is negligible. The equipotential lines still cross the aquifer vertically and meet its lower boundary perpendicularly. They are not refracted (except at the extremities of the flow net) and are vertical over most of the $K = 1$ layer down to the base of the model. Thus, the lower boundary of the $K = 10$ layer has the same effect as an impermeable boundary.

It is concluded that when one is designing models of groundwater basins, there is some depth in regions of reasonably horizontal sedimentation below which the equipotential lines will remain vertical. On the basis of our hypothetical two-dimensional models, this depth appears to be the lower boundary of the deepest aquifer whose permeability significantly exceeds that of underlying beds. To be on the safe side, preliminary studies should probably begin with a greater basin depth than would seem necessary. If one finds vertical equipotentials as suggested by the above, the basin depth can be limited accordingly.

The twenty-one potential nets presented in this paper have been chosen from a much larger number that appear in the original study [*Freeze,* 1966]. Such idealized hypothetical models provide an excellent insight into the nature of groundwater flow, but it must be recognized that it would take a much larger number of models to cover even a fraction of all of the possible combinations of water-table configurations and subsurface permeability variations. In the ultimate practical use of our method, a mathematical model must be designed for each individual groundwater basin under investigation.

CONCLUSIONS

1. The factors that affect steady-state regional groundwater flow patterns within a nonhomogeneous, anisotropic basin are: (a) 'depth : lateral extent' ratio; (b) water-table configuration; and (c) the stratigraphy and resulting subsurface variations in permeability.

2. It is possible to study the influence of these factors by utilizing digital computer solutions to appropriately designed mathematical models.

3. The presence of a major valley will tend to concentrate discharge in the valley. Where the regional water-table slope is uniform, the entire upland area is a recharge area. In hummocky terrain, numerous sub-basins will be superposed on the regional system.

4. The presence of a buried aquifer of significant permeability will have a profound effect on regional groundwater flow. It acts as a highway that transmits water to the principal discharge area and affects the magnitude and position of the recharge areas. It can also have a significant effect on the extent and flow capacity of sub-systems within the regional basin. Buried aquifers need not outcrop to produce artesian flow conditions.

5. Certain nonhomogeneous basins may have potential fields that are indistinguishable from those of the homogeneous case.

6. Stratigraphic pinchouts at depth can create recharge or discharge areas where they would not be anticipated on the basis of the water-table configuration.

7. There is some depth in regions of reasonably horizontal sedimentation below which equipotential lines remain vertical. This condition allows the effective depth to the basal impermeable boundary to be selected when designing a mathematical model.

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