Theoretical Analysis of Regional Ground Water Flow
3. Quantitative Interpretations

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Abstract. The natural basin yield of a ground water basin can be calculated from a quantitative analysis of the flow net obtained from a digital computer solution to a numerical mathematical model of the basin. The natural basin yield is an unique property of the basin and can be considered as a measure of the ground water recharge to the basin and a conservative estimate of the safe yield. Rates of ground water recharge and discharge vary from place to place throughout the areal extent of a basin; a quantitative analysis can be used to determine the positions of concentration. Quantitative interpretation of ground water flow nets can play an important role in the calculation of basin-wide water balances due to the interrelationships between ground water recharge and infiltration at one end of the flow system and ground water discharge, evapotranspiration, and stream baseflow at the other. (Key words: Ground water; computers, digital; drainage basin characteristics)

QUANTITATIVE FLOW NETS

In two preceding papers [*Freeze and Witherspoon*, 1966; *Freeze and Witherspoon*, 1967] the authors have developed a method of analyzing regional ground water flow based on digital computer solutions to numerical mathematical models. The method is applicable to three-dimensional nonhomogeneous anisotropic ground water basins with any water table configuration. The principal directions of anisotropy must, however, coincide with the coordinate directions. In the preceding papers the results were presented in the form of potential nets and *qualitative* flow nets in which the streamlines indicated the direction of flow but did not have quantitative significance. It *is* possible, of course, to construct quantitative flow nets from the potential patterns representing regional ground water flow [*Harr*, 1962].

Figure 1 shows a two-dimensional quantitative flow net for a hypothetical ground water basin. The basin is bounded on each side by imaginary vertical impermeable boundaries [*Freeze and Witherspoon*, 1966] and on the bottom by a basal impermeable boundary. The upper boundary of flow is the water-table. The basin has a dimensionless length of s and a depth of 0.1 s measured at the right-hand side. The configuration of the water-table shows the effect (from left to right) of a major valley, a steep valley flank, and a constant gentle regional upland with a slope of about 0.8%. The total relief on the water-table is $0.0167s$, half of which is taken up in the valley flank. The subsurface stratigraphy consists of a sloping formation of low permeability bounded above and below by higher permeability layers. The permeability ratio is 10:1. Each formation is homogeneous and isotropic. The dashed lines represent equipotential lines, and the solid lines represent flow lines. The flow lines are drawn orthogonal to the equipotential lines and in such a way that curvilinear squares are formed throughout the $K = 10$ region. Owing to the refraction at the permeability interfaces, curvilinear rectangles 10 times as wide as they

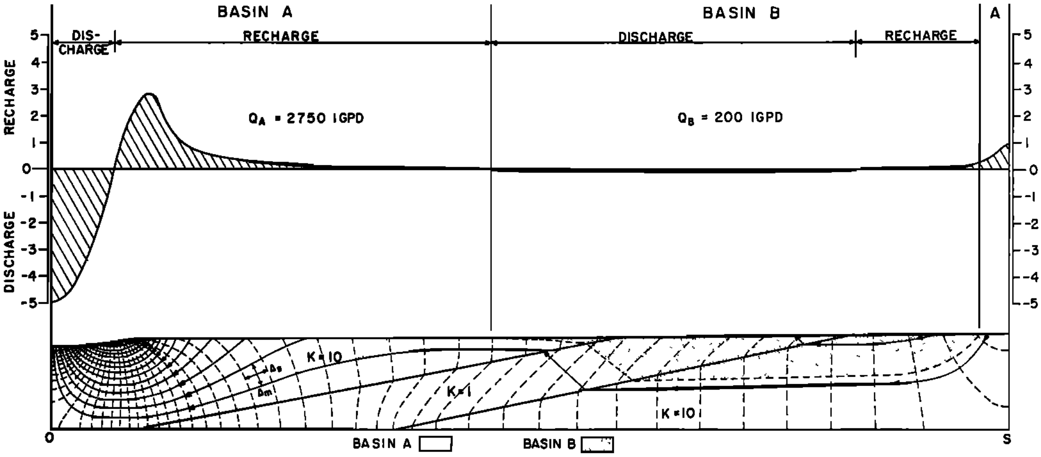

Fig. 1. Quantitative flow net and recharge-discharge profile for a two-dimensional, nonhomogeneous, isotropic composite ground water basin.

are long will result in the $K = 1$ layer. The flow net was constructed graphically from the basal boundary up; partial squares result at the upper boundary.

Having defined the flow net, one can calculate the flow through each channel from Darcy's law

$$Q = K\,(\Delta\varphi/\Delta s)\,\Delta m\,w \qquad (1)$$

where

Q = flow through a segment of the flow net;
K = permeability;
$\Delta\varphi$ = drop in hydraulic head between equipotential lines;
Δs = interval between equipotential lines (see Figure 1);
Δm = width of the segment of the flow net perpendicular to direction of flow (see Figure 1);
w = thickness of the flow system perpendicular to the plane of the diagram.

For the square portion of the net, $\Delta s = \Delta m$, and considering a unit thickness of the system ($w = 1$) we are left with

$$Q = K\Delta\varphi \qquad (2)$$

The discharge in each flow channel remains constant throughout its length, and the discharge in all flow channels is equal. One can therefore determine the total discharge through the ground water basin by summing the quantities of flow in the individual channels.

In Figure 1, the water-table configuration and geologic conditions give rise to two separate flow systems: a local flow system which is shallow but of large lateral extent and a larger regional system. The local system is superposed on the regional system in a way that could hardly have been anticipated by other means than that of a theoretical model. If a *ground water basin* is defined as a 'three-dimensional closed system which contains the entire flow paths followed by all water recharging the basin' [*Freeze and Witherspoon*, 1967], then it is clear that the single topographically defined basin represented by the model is actually a *composite basin* consisting of two separate ground water basins (denoted as Basin A and Basin B on Figure 1).

Using equation 2 one can easily calculate the quantity of flow in each basin. For s = 60,000 ft, the total relief is 1000 ft, and since there are 50 increments of potential, $\Delta\varphi$ = 20 ft. Assuming permeabilities of 10 and 1 IGPD/ft² (imperial gallons per day per square foot), the flow channel discharge is 200 IGPD (per foot of thickness of the flow system perpendicular to the diagram). By simply counting the flow channels in the two basins we arrive at the following discharges:

$$Q_A = 2750 \text{ IGPD}$$
$$Q_B = 200 \text{ IGPD}$$

Any two permeabilities in the ratio of 10:1 will give the same flow net for this configuration of water-table and geology, but the values of Q_A and Q_B will vary in direct proportion to the permeability values.

The presence of anisotropy renders a quantitative analysis more difficult. For simple geometric domains such as uniformly layered mediums, recourse can be made to the method of the transformed section [*Harr*, 1962]. For complex cases, more representative of true geologic configurations, certain quantitative information can be extracted from flow pattern results using the same principle, but methods to delineate complete quantitative flow nets in such cases are not yet available. An example of the type of quantitative information that can be determined from potential patterns for a two-dimensional, irregular, nonhomogeneous anisotropic basin is provided later in this paper.

NATURAL BASIN YIELD

For the purposes of this paper, the quantity of flow through an undeveloped basin under natural conditions is defined as the *natural basin yield*. The quantities calculated in the preceding section represent the natural basin yields of the two component basins of Figure 1.

Under the assumption of a steady-state water-table [*Freeze and Witherspoon*, 1966], the value of the natural basin yield represents a constant discharge that does not change with time. It is important to recall that our definition of a 'steady-state' water-table does not deny the existence of water-table fluctuations. It *does* state that their effect on the flow patterns will be small if (1) The zone of fluctuation of the water-table is only a small percentage of the total saturated depth of the ground water basin; and (2) The relative configuration of the water-table remains the same throughout the cycle of fluctuations. If these two conditions are satisfied, the small uniform fluctuations in the water-table will not result in any significant change in the nature of the flow pattern or in the quantity of flow through the basin. The natural basin yield is relatively stable within the normal variations of precipitation; it will not fluctuate significantly with time and therefore represents a unique property of the basin.

The fact that small-scale cycles of rainfall result in wet and dry periods through the year will serve to cause the fluctuations in the water-table, which we have pointed out can often be approximated by a steady-state average. The effect of a long term increase in the total annual precipitation would be a more permanent raising of the water-table, which would significantly increase the ground water storage of the basin but would only slightly increase the natural basin yield. As stated above, we assume that the average slope of the water-table does not change appreciably. The maximum possible natural basin yield would occur if the water-table were everywhere coincident with the ground surface. The natural basin yield can be considered as a measure of the quantity of water which a given basin can accept and is therefore a measure of the ground water recharge to the basin. The ultimate effect of a long term increase in annual precipitation would therefore be that a greater proportion of the increased rainfall would become surface runoff.

In defense of requirement (1), it can be noted that in the semi-arid Canadian prairies the usual annual fluctuations in the water-table are of the order of 5 ft and are always less than 10 ft. The maximum difference in elevation of the water-table between an extended period of wet years and dry is of the order of 20 ft, and the depths of ground water basins are 300–2000 ft. Under these conditions, if requirement (2) is not violated, the concept of natural basin yield is valid.

If either of the conditions (1) or (2) is violated, then the methods of this chapter must be altered. It may be necessary to calculate the natural basin yield on a monthly basis, for example, using twelve different water-table configurations representing the fluctuating position of the water-table throughout the year. For example, *Meyboom* [1966] has shown that in the vicinity of a willow ring in hummocky moraine, the water-table undergoes transient fluctuations such that the enclosed temporary slough is a recharge area at certain times of the year and a discharge area at others. He has prepared a water balance for the slough that required consideration of 29 separate time intervals throughout the year. A mathematical model analysis of such a flow system would presumably require a similar number of steady state runs. In such cases, a transient mathematical model might prove more efficient.

The *safe yield* of a ground water basin is the amount of water that can be withdrawn from it annually without producing an undesired result [*Todd,* 1959]. The undesired result may be depletion of the resource, impairment of the quality of the water, or the creation of an economic or legal problem. Considering the safe yield from a strictly quantitative point of view, it is logical to inquire as to the relation between the *safe yield* and the *natural yield* of a basin.

The safe yield and the natural yield are not identical; the natural basin yield refers to a virgin basin that has not undergone ground water development. The introduction of a major well field will change the conditions governing the existing flow pattern by creating a cone of depression in the water-table in the case of an 'unconfined aquifer' or lessening the potential at depth as would be the case in the development of a 'confined aquifer.' The effect of this artificial discharge from the basin will be to create a new flow pattern from which a new basin yield can be derived for that stage of development. Further development will result in more changes which usually tend to increase the ground water yield. There is some optimum development for the basin that maximizes the safe yield.

When the initial ground water development of a virgin basin is contemplated, calculation of the natural basin yield as determined from the results of a mathematical model analysis will provide a conservative estimate of the basin ground water yield that could be tapped in the initial development. When the effects of this initial development on the previously existing flow pattern have been determined, a new model can be constructed that includes the effects of the well field. This model can then be used to estimate the expected basin yield from further ground water development.

Used in this stepwise fashion, the steady-state mathematical model can be useful in estimating the order of magnitude of the basin safe yield. For more detailed analyses, transient mathematical models have many advantages, but their use for large three-dimensional basins is restricted at present by computer limitations.

The concept of natural basin yield has ramifications in the basin-wide development of ground water resources. For example, consider the two component basins of Figure 1. Each has approximately the same surface area, yet Basin A has a natural basin yield nearly 14 times that of Basin B. The optimum location for ground water development would appear to be in the high permeability layer within Basin A.

This method may have only limited application to basins that are already heavily developed, such as those around the large metropolitan centers of the United States and Canada, but it can be very useful in planning the development of the many near-virgin basins that abound in the American west and the Canadian prairies and northlands.

RECHARGE-DISCHARGE PROFILE

Above the flow net in Figure 1 is a plot of the quantitative distribution of recharge and discharge. Such a plot will be designated as a *recharge-discharge profile.* The crosshatched areas above the center line represent recharge; those below represent discharge. The profile is constructed by drawing a column upward or downward from the zero line directly above the intersection of each flow channel with the water-table, such that the areas of all the columns are equal. In zones of concentrated recharge or discharge the distance between flowlines intersecting the water-table will be small, and the column will be tall and narrow. Where flow to or from the water-table is less concentrated, the flowlines will be distant from one another and the column wide and flat. The series of columns, which have the appearance of a composite bar graph, are then replaced by a smooth curve such as that shown in Figure 1. The total areas above and below the line must be equal within a given basin. The scales shown at either end of the profile are arbitrary but would have units of the form IGPD/ft^2, or simply ft/day. The recharge-discharge profile is thus a graphical representation of the recharge or discharge rate at any point on the cross section. The form of this plot was suggested to us by comments made by *Davis* [1963] in regard to a paper by *Tóth* [1962].

One notes that concentrations of recharge and discharge occur at points that generally could not have been anticipated. We believe that it is probably impossible to conceive of a configuration of water-table and stratigraphy that will not give rise to concentrations of discharge and recharge at various points on the surface of a basin. *Klute et al.* [1965] used a similar diagram and noted similar concentrations in their analysis of steady-state flow in a saturated inclined soil slab.

In Figure 1 the entire discharge from Basin A takes place in the major valley and on the lower half of the valley flank. Recharge is *concentrated* on the constant regional slope just above the valley flank. This recharge area extends laterally as far as the hinge-line separating Basins A and B, but the magnitude of recharge over a large portion of this area is small. Here we have an example of a recharge area of large lateral extent but of low rate of inflow. A second area where recharge to Basin A is slightly concentrated occurs at the right-hand topographic divide. The rate of recharge and discharge in Basin B is low over the entire lateral extent of the basin, a fact which is in correspondence with the low natural basin yield of this basin.

The recharge-discharge regime can be an important indicator of the other components of the hydrologic cycle. For example, in discharge areas where the water-table is kept high by the upward rising ground water, evapotranspiration rates will be high. Similarly, the ground water component of surface runoff will be controlled by the existing ground water flow pattern. For example, in Basin A (Figure 1) where the entire discharge occurs in the major valley, the rate of discharge into the valley must equal the natural basin yield. For $s = 60{,}000$ ft, we showed that $Q_A = 2750$ IGPD per foot thickness perpendicular to the plane of the diagram. Noting that the discharge area covers 3960 ft laterally, the average rate of discharge over the discharge area is computed to be 0.11 ft/day. In the Canadian prairies a usual figure for evapotranspiration is 0.02 ft/day, leaving 0.09 ft/day per longitudinal foot, or a total of 2250 IGPD for baseflow to a stream flowing perpendicular to the cross section. If such a stream had a tributary flowing across Basin A from right to left parallel to the cross section of Figure 1, one would expect the stream to be influent while it traverses the recharge area but effluent as it crosses the discharge area, with the ground water component of the streamflow increasing downstream.

In a small basin over long periods of time, it is likely that the quantity of rain falling on the recharge area will be roughly equal to that falling on the discharge area. The behavior of the rain upon reaching the ground, however, will be influenced by the existing ground water flow pattern. In a recharge area, a downward potential gradient exists that would tend to take a larger percentage of the moisture surplus into the ground than in a discharge area. Water which infiltrates to the water-table in a discharge area can only be transmitted back to the surface again by an agent of discharge, such as evapotranspiration, when conditions permit. In general, constantly changing climatic conditions at the surface cause rapid timewise variations in the rate of infiltration and evaporation above the water-table. The unsaturated zone acts as a buffer that attenuates these effects, so that recharge and discharge rates below the water-table remain relatively constant [*Freeze*, 1967*a*].

Since ground water flow patterns exert such an important influence on the quantity and areal concentration of infiltration, evapotranspiration, and surface runoff, it is clear that theoretical quantitative flow nets derived from the use of mathematical models can be an important tool in the calculation of basin-wide water balances.

It should be noted that the effects of bank storage, a major factor in the fluctuation of baseflow with time, cannot be taken into account by a single steady-state model. Once again, a sequence of models representing the changing conditions would be necessary.

The analysis of ground water recharge can lead one into a vicious circle. We have noted that the existing water-table configurations, which controls the nature of the ground water flow pattern, will influence the quantity of ground water recharge. But it is also true that the nature and amount of rainfall will control, to a certain degree, the configuration of the water-table. In this study we have assumed a fixed location of the water-table and developed the recharge and discharge patterns. Future studies should consider the effect of climatic patterns on water-table configurations. These studies must necessarily include consideration of the unsaturated zone and will require treatment of transient phenomena. A one-dimensional approach has just been completed [*Freeze,* 1967*a*].

A TWO-DIMENSIONAL FIELD EXAMPLE

Figure 2 shows a two-dimensional flow pattern analysis for the Gravelbourg aquifer in south central Saskatchewan, Canada. Figure 2A is the flow pattern as determined by field evidence [*Freeze,* 1967*b*]; Figure 2B is the flow pattern and recharge-discharge profile as determined from mathematical model results. The location of the two-dimensional cross section was chosen so that it is representative of the regional stratigraphy and is in a position parallel to the direction of flow in the Gravelbourg aquifer as determined from a regional piezometric surface map.

The cross section on Figure 2A shows uplands to the north and south rimming a gently sloping plain of large lateral extent. The plain is incised by two creeks: Notukeu Creek and Wiwa Creek. The valley of the latter is the major topographic low in this area.

The geological formations (Figure 2A) are Pleistocene in age and glacial in origin. A thin veneer of glacial lake clay overlies 50–100 feet of glacial till, which in turn overlies a stratified drift deposit consisting of an upper silty clay phase and a lower sand phase. The sand phase of the lower stratified drift is termed the Gravelbourg aquifer. The aquifer provides a source of water for the town of Gravelbourg and is tapped extensively by the rural residents of the Gravelbourg plain. All the glacial deposits are unconsolidated. They are underlain by a thick section of compacted clay shales of the Bearpaw formation of Cretaceous age. The permeability of the Bearpaw shale is several orders of magnitude lower than that of the Gravelbourg aquifer, and its upper surface can safely be considered as the base of the regional ground water flow system in this area. The position of the water-table closely follows the topography as shown in Figure 2A.

The flow pattern shown in Figure 2A was determined entirely on the basis of field evidence and on the interpretation of field measurements. The equipotential lines were drawn from their known point of intersection with the water-table to their known positions within the aquifer as taken from a piezometric surface map prepared from the available well records. The existence of horizontal flow in the aquifer was confirmed by a piezometer instal-

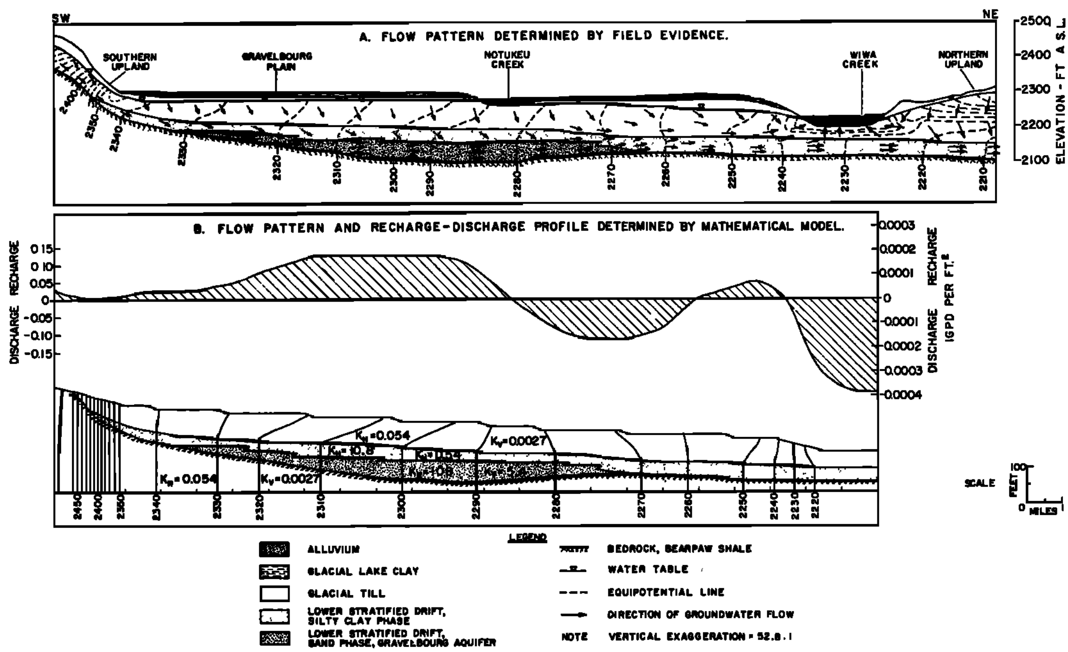

Fig. 2 Comparison of two-dimensional flow pattern for Gravelbourg aquifer, Saskatchewan, Canada, as determined by field evidence and by mathematical model.

lation at the town of Gravelbourg near the center of the Gravelbourg plain. The piezometer installation also showed leakage from the overlying silty clay phase of the lower stratified drift into the aquifer, and slight leakage from the aquifer into the underlying Bearpaw shale. Wiwa Creek Valley was mapped as a discharge area on the basis of surficial evidence in the form of saline soils, phreotophytic vegetation [*Meyboom*, 1967], high water-tables, and flowing wells. The directions of ground water flow shown by the arrows on Figure 2A are drawn with regard to the vertical exaggeration of the plot of 52.8:1 [*van Everdingen*, 1963]. By inference, the formations are considered to be isotropic. A complete description of the methodology and results of the field hydrogeological investigation of the Gravelbourg area has appeared elsewhere [*Freeze*, 1967*b*].

The two-dimensional mathematical model (Figure 2B) used to simulate flow conditions in the Gravelbourg area consists of a nodal mesh with 48 nodes in the horizontal direction and 31 nodes in the vertical direction. The horizontal nodal spacing is 2640 feet, the vertical nodal spacing 12.5 feet. The total width of the model is 124,080 feet (23.5 miles); the total depth, 375 feet. The model is bounded on both sides and on the bottom by impermeable boundaries [*Freeze and Witherspoon*, 1966]. The water-table is simulated in the position determined by field measurement. The geological formations are also built into the model in their actual configuration with the exceptions of the surficial glacial lake clay and the alluvial deposits that were omitted from the model. It is felt that these formations exert little influence on the large-scale, steady-state, saturated flow system in that they both lie almost entirely above the water-table. The lake clay undoubtedly *does* have considerable influence on the transient, unsaturated infiltration that gives rise to ground water recharge [*Freeze*, 1967*a*].

Figure 2A, which is taken from *Freeze* [1967*b*], shows underflow beneath Wiwa Creek Valley through the silty clay phase of the lower stratified drift. The field evidence for this phenomenon was inconclusive. On the basis of preliminary mathematical models that extended farther to the northeast than Figure 2A, it was found that where the northern upland has relief such as that shown in Figure 2A, Wiwa Creek Valley is a sufficient topographic low to create an imaginary impermeable boundary extending to the base of the model. For this reason the model shown in Figure 2B is terminated beneath Wiwa Creek. Underflow beneath Wiwa Creek Valley undoubtedly does occur in those parts of the basin where the northern upland has low relief or is nonexistent.

The major problem associated with the design of the mathematical model was the assignment of permeability values to the various formations. From the geological evidence, it was concluded that different permeability ranges should be used for: (1) the Gravelbourg aquifer; (2) the silty clay phase of the lower stratified drift; and (3) the glacial till and Bearpaw shale. On the basis of grain size analyses and permeability measurements on similar deposits by other workers in the Canadian prairies, it appeared that the ratio of permeabilities for these ranges should be in the order 1:0.1:0.0005. To obtain one absolute value to which the other relative values could be tied, a pump test was run on the Gravelbourg aquifer. The horizontal permeability thus obtained was 108 IGPD/ft^2 [*Freeze*, 1967*b*].

Preliminary models with isotropic formations failed to produce realistic flow patterns. A horizontal: vertical factor of anisotropy of 20:1 in all formations was finally chosen, and this resulted in the horizontal (K_h) and vertical (K_v) permeabilities shown on Figure 2B.

The mathematical model required 3360 iterations to arrive at a tolerance of 0.00076 from a fairly coarse set of initial values for φ. The problem consumed 31 minutes of computer time (IBM 7094).

The equipotential lines determined by the mathematical model (Figure 2B) are in good agreement with those determined from field evidence (Figure 2A), particularly in the upstream half of the basin. The gradient within the aquifer as determined by the model is almost identical to that measured in the field. A discharge area that is not evident in the field shows up between Notukeu Creek and Wiwa Creek on the mathematical model. This could result from an inaccurate simulation of the thickness or permeability of the lower stratified drift in the downstream portion of the model.

The directions of ground water flow have been omitted from Figure 2B. They must be drawn with regard to the vertical exaggeration of the plot and the presence of anisotropy. They are not perpendicular to the equipotential lines, except at those points in the model where the equipotential lines are horizontal or vertical, as is the case within the Gravelbourg aquifer.

A recharge-discharge profile has been constructed above Figure 2B. Owing to the presence of irregular nonhomogeneous domains and anisotropic formations, the method of constructing the profile outlined earlier in this paper is inapplicable. Instead, the following approach was followed:

1. The direction of ground water flow at the water-table was determined at many points along the water-table configuration. This was done by using the method of the transformed section for the portion of model above the upper boundary of the lower stratified drift (i.e., within the glacial till formation). For this upper part of the model, the vertical exaggeration of the plot was reduced from 52.8:1 to 4.47:1, thus creating the equivalent isotropic potential configuration for the 20:1 anisotropy factor used for the glacial till.

2. The permeability value in the direction of ground water flow was determined from the permeability ellipse of the glacial till.

3. The hydraulic gradient was determined from the print-out of the mathematical model results.

4. Using Darcy's law, the rate of ground water recharge or discharge was calculated for each point and plotted on the recharge-discharge profile. Where the equipotential lines hit the water-table at right angles, flow is parallel to the water-table, and little or no recharge results.

Two sets of units are noted for the profile: IGPD/ft^2 at the right-hand end, and in./yr at the left-hand end. The maximum rate of recharge over the basin is 0.0002 IGPD/ft^2 (0.15 in./year); the minimum values of 0.0 occur at the hinge lines. The absolute values plotted on the profile are based on relating the permeabilities used in the glacial till to the absolute value of horizontal permeability measured in the Gravelbourg aquifer.

It is possible to determine the natural basin yield by integrating the area above or below the zero line on the recharge-discharge profile. The natural basin yield for this two-dimensional cross section (Figure 2B) is 12.2 IGPD (per foot of thickness of the flow system perpendicular to the diagram).

It is also possible to obtain an estimate of that portion of the natural basin yield that is flowing through the Gravelbourg aquifer. From Figure 2B we note that the prevailing gradient over most of the aquifer is about 5 ft/mile and that the average thickness is about 40 ft. Using equation 1, we have

$$Q = [(108)(5)(40)(1)]/5280 = 4.1 \text{ IGPD}$$

Therefore, on the basis of this particular cross section, about one-third of the total natural basin yield is flowing through the Gravelbourg aquifer.

A THREE-DIMENSIONAL FIELD EXAMPLE

The Good Spirit Lake drainage basin in east central Saskatchewan, Canada, is being studied as a 'representative basin' in Canada's International Hydrologic Decade program. Emphasis in the study is on improved techniques of measuring the ground water component of the hydrologic balance. To this end a three-dimensional numerical mathematical model was designed to simulate the ground water flow patterns within the basin.

The methods described in the initial paper in this series [*Freeze and Witherspoon*, 1966] were used. The water-table configuration and permeability values were determined by field measurement. A 30 × 46 × 20 array of nodal points was used, and a converged solution with a tolerance of 0.01 ft of hydraulic head was reached after 93 iterations and 1 hour and 40 minutes of computer time (IBM 7094). The results of the model are in excellent agreement with piezometer measurements and field mapping [*Freeze*, 1967*c*]. The model provides a three-dimensional potential configuration from which flow directions can be determined and to which the quantitative interpretations described in this paper can be applied.

From a three-dimensional analysis, it is possible to prepare a recharge-discharge map showing the areal variations in the rates of recharge and discharge throughout the basin. This can be considered as the three-dimensional form of the recharge-discharge profile. Such a map has been prepared for the central sub-basin of the

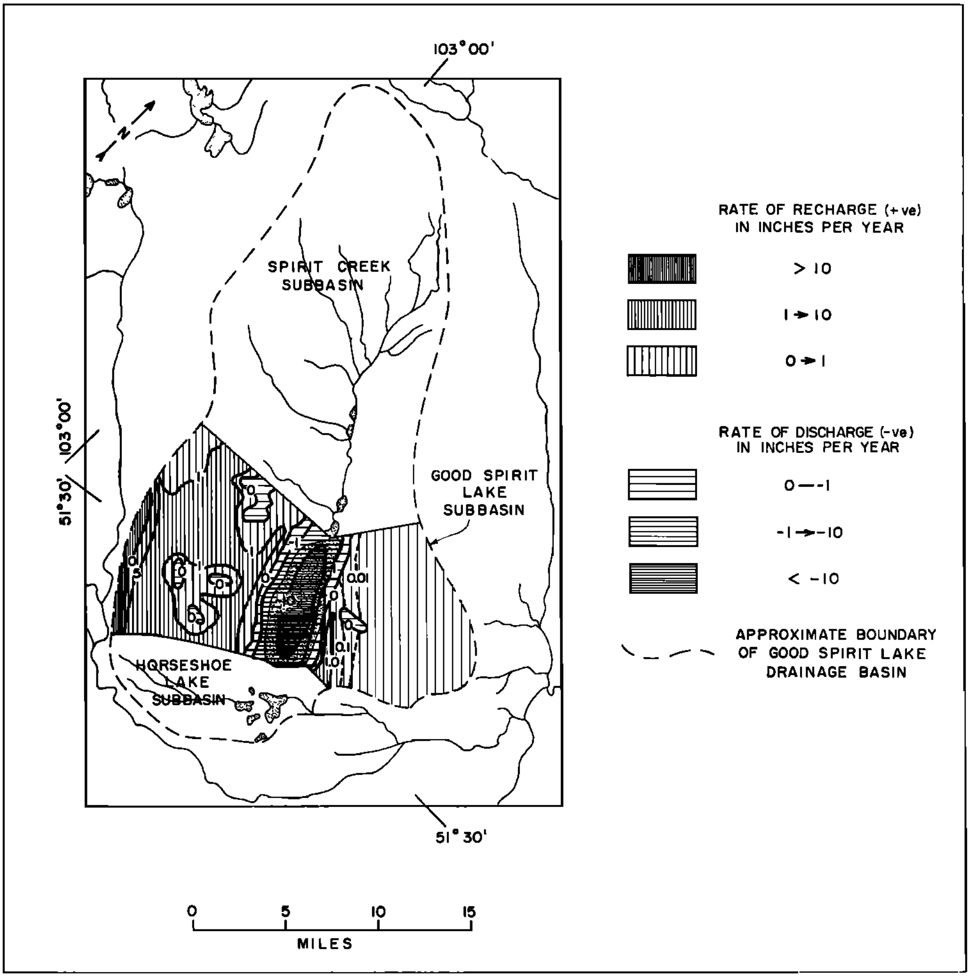

Fig. 3. A recharge-discharge map from a three-dimensional field example, Good Spirit Lake drainage basin, Saskatchewan, Canada (after *Freeze,* 1967c).

Good Spirit Lake drainage basin. It is reproduced in Figure 3 (after *Freeze,* 1967*c*).

The sub-basin can be divided into three zones: a recharge area west of Good Spirit Lake with high rates of ground water recharge, a recharge area east of Good Spirit Lake with low rates of ground water recharge, and a central discharge area in the vicinity of the lake itself. The surficial deposit west of the lake is outwash sand and gravel; east of the lake it is glacial till. The five small discharge areas that occur within the boundaries of the recharge areas are local flow systems coinciding with the locations of permanent sloughs.

The natural basin yield of the Good Spirit Lake sub-basin is 12,000 acre-ft/yr. Of this total, 96% enters the flow system in the sand-gravel plain west of the lake. Relationships between the other components of the water balance, the recharge and discharge rates, and the natural basin yield are discussed by *Freeze* [1967*c*].

CONCLUSIONS

1. It is possible to construct quantitative flow nets representing regional ground water flow from potential patterns obtained as digital computer solutions to numerical mathematical models.

2. The natural basin yield is defined as the

quantity of flow through an undeveloped ground water basin under natural conditions. It is a consequence of the existing potential field which, in turn, is controlled by the water-table configuration and the geometry and value of the permeability contrasts created by the geologic configuration. It represents a unique property of the basin which is relatively stable within the normal variations in precipitation. It can be considered as a measure of the quantity of water that a given basin can accept as recharge. The natural basin yield provides a conservative estimate of the safe yield of the basin.

3. The rate of ground water recharge and discharge varies from place to place throughout the area extent of a basin. The positions of concentration of recharge and discharge can best be delineated with the aid of a recharge-discharge map in a three-dimensional analysis.

4. The nature of the ground water flow pattern is an important indicator of the quantity and areal concentration of the other components of the hydrologic cycle, in particular of infiltration, evapotranspiration, and surface runoff.

5. The use of mathematical models to derive theoretical quantitative flow nets can be an important tool in the calculation of basin-wide water balances.

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