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Comment on "An Investigation of the Relationship Between Ponded and Constant Flux Rainfall Infiltration" by A. Poulovassilis et al.

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Introduction

Poulovassilis et al. [1991] deal with the important problem of switching from constant flux (i.e., atmosphere controlled) infiltration to ponded (i.e., soil controlled) infiltration which is a vital factor in any realistic scheme of soil moisture accounting. The authors conclude that the profile developed at the time $t = T$, when the surface becomes saturated, under constant flux conditions is identical to the profile developed at the time $t = t_c$, when the infiltration rate is equal to the rainfall flux, under instantaneous ponding conditions. The results obtained from the analysis of this switching problem at the Centre for Water Resources Research in University College Dublin indicate that this general theoretical statement is approximately but not exactly true [*Sander et al.*, 1986; *Kühnel*, 1989; *Kühnel et al.*, 1986, 1990; *Wang and Dooge*, 1993].

The solutions of equation (1) of *Poulovassilis et al.* [1991] for rainfall infiltration $\theta_R(z, t)$ and for flooding infiltration $\theta_F(z, t)$ are different in functional form because of the difference in boundary conditions. This can be shown analytically for all the special forms of the relationships $K(\theta)$ and $H(\theta)$ for which closed-form solutions exist and can be shown numerically for empirical data on real soils. It is possible to make two solutions coincide at the surface both for concentration, i.e., $\theta_R(0, T) = \theta_F(0, t_c) = \theta_{\text{sat}}$ and for flux, i.e., $-D\partial\theta_R/\partial z|_{0,T} = -D\partial\theta_F/\partial z|_{0,tc} = q - K$, but the resulting profiles will not be identical throughout the soil column.

In this comment, the hypothesis of identical profiles for all cases suggested by *Poulovassilis et al.* [1991] is tested against both analytical and numerical solutions, and their theoretical arguments in support of the hypothesis are examined. If the hypothesis were exact, it would hold for all cases both idealized and actual. We examine the hypothesis for four separate cases, three idealized cases for which analytical solutions are available and one of the actual soils for which the authors present a numerical solution. The hypothesis of exact profile matching is seen to fail in both the idealized cases and the actual soil since the matching of the surface fluxes at ponding gives rise to a difference in volume of cumulative infiltration. If the alternative approach of matching the volumes is used, there is an analogous failure of the hypothesis of identical profiles since it can be shown that there is for that case a discontinuity in the surface flux at ponding. If an attempt is made to match both the flux and the volume by adjusting both the time of constant flux (T) and the time of flooded infiltration (t_c), the condition of surface saturation will not be met for the rainfall case.

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Analytical Solutions for Constant Diffusivity

We consider a semi-infinite homogeneous and initially dry soil column. Closed-form analytical solutions are available for both flux and concentration boundary conditions for a number of functions of hydraulic diffusivity $D(\Theta)$ and hydraulic conductivity $K(\Theta)$ [see *Kühnel et al.*, 1990, Table 1]. The hypothesis of *Poulovassilis et al.* [1991] of identical profiles can be tested by using these solutions and comparing the volume of water under the soil moisture profile at $t = T$ for the constant surface flux solution (R profile) and the volume of water under the profile at $t = t_c$ for the saturated surface concentration (F profile). These respective volumes are denoted in this discussion by $W_R(T)$ and $W_F(t_c)$.

For the special case of constant diffusivity $D(\Theta)$ and constant conductivity $K(\Theta)$, the general nonlinear equation for soil moisture movement given by equation (1) of *Poulovassilis et al.* [1991] reduces to the linear diffusion equation whose solution is well known. Under the condition of constant rainfall flux q at the surface, it can be shown that the volume of soil moisture $W_R(T)$ at the instant of ponding is given by

$$W_R(T) = \frac{\pi}{4} \frac{D\Theta_s^2}{q - K} \tag{1}$$

Under the condition of instantaneous ponding, the volume of soil moisture $W_F(t_c)$ at the time the infiltration equals constant rainfall flux q is given by

$$W_F(t_c) = \frac{2}{\pi} \frac{D\Theta_s^2}{q - K} \tag{2}$$

The ratio of these two volumes is seen to be constant since

$$\frac{W_F(t_c)}{W_R(T)} = \frac{8}{\pi^2} = 0.811 \tag{3}$$

Analytical solutions are also available for the special cases of constant diffusivity and linear or quadratic conductivity. For constant $D(\Theta)$ and quadratic $K(\Theta)$, it can be shown [*Philip*, 1974; *Sander et al.*, 1986] that for the concentration boundary condition

$$W_F(t_c) = \rho_1(q/K_s) \frac{D\Theta_s^2}{K_s} \tag{4}$$

where $\rho_1(q/K_s)$ is a complex function of q/K_s obtained by equating the surface flux to the rainfall rate. Similarly, it can be shown [*Clothier et al.*, 1981; *Sander et al.*, 1986] that for the constant flux boundary condition

$$W_R(T) = \rho_2(q/K_s) \frac{D\Theta_s^2}{K_s} \tag{5}$$

TABLE 1. Ratios of $W_F(t_c)/W_R(T)$ for Constant Diffusivity and Quadratic Conductivity Solution

$(q - K_s)/K_s$	$W_R(T)K/D\Theta_s^2$	$W_F(t_c)K/D\Theta_s^2$	$W_F(t_c)/W_R(T)$
10^{-4}	8.220	7.633	0.932
10^{-3}	6.056	5.542	0.915
10^{-2}	3.985	3.512	0.881
10^{-1}	1.980	1.694	0.856
10^{0}	0.553	0.455	0.823
10^{1}	0.075	0.061	0.812
10^{2}	0.0079	0.0064	0.811

TABLE 2. Ratios of $W_F(t_c)/W_R(T)$ for Fujita Diffusivity and Constant Conductivity Solution

ν	$W_R(T)(q - K)/\bar{D}\Theta_s^2$	$W_F(t_c)(q - K)/\bar{D}\Theta_s^2$	$W_F(t_c)/W_R(T)$
0.0	0.785	0.637	0.811
0.1	0.803	0.655	0.816
0.5	0.886	0.749	0.845
0.9	0.989	0.920	0.930
1.0	1.000	1.000	1.000

where $\rho_2(q/K_s)$ is a complex function of q/K_s obtained by equating the surface concentration to saturation. The ratio of the two volumes given by (4) and (5) is clearly a function of q/K_s only. The variation with q/K_s of the ratio of the two volumes $W_F(t_c)/W_R(T)$ is shown in Table 1.

ANALYTICAL SOLUTIONS FOR FUJITA DIFFUSIVITY

Closed-form analytical solutions are also available for constant $K(\Theta)$ and Fujita $D(\Theta)$ defined by

$$D = \frac{(1 - \nu)\bar{D}}{(1 - \nu\theta)^2} \tag{6}$$

where θ is reduced soil moisture content, $\bar{D}$ is average hydraulic diffusivity, and ν is a shape parameter ranging from $\nu = 0$, which corresponds to constant $D(\Theta)$, to $\nu = 1$, which corresponds to the delta $D(\Theta)$ function [*Fujita*, 1952]. In fitting (6) to the hydraulic diffusivity variation for real soils the value of parameter ν is found to vary between about 0.1 for heavy clays and 0.9 for light sands.

It can be shown that for the concentration boundary condition [*Fujita*, 1952; *Sander et al.*, 1986] this formulation gives a relationship of the type

$$W_F(t_c) = \phi_1(\nu)\,\frac{\bar{D}\Theta_s^2}{q - K} \tag{7}$$

where $\phi_1(\nu)$ is a complex function of ν obtained by equating the surface flux to the rainfall rate. Similarly, it can be shown [*Knight and Philip*, 1974; *Sander et al.*, 1986] that for the constant flux boundary condition

$$W_R(T) = \phi_2(\nu)\,\frac{\bar{D}\Theta_s^2}{q - K} \tag{8}$$

where $\phi_1(\nu)$ is a complex function of ν obtained by equating the surface concentration to saturation. Comparison of (7) and (8) shows that the ratio of the two volumes depends only on ν. The variation with ν of ratio of the two volumes $W_F(t_c)/W_R(T)$ is shown in Table 2. For $\nu = 0$ the ratio is the same as (3), and for $\nu = 1$ the volumes are identical.

Both Tables 1 and 2 show that $W_F(t_c)$ is always less than $W_R(T)$, and thus the hypothesis of identical profiles cannot be sustained, except for the single case when $D(\Theta)$ is a delta function for which identical rectangular profiles are postulated from the beginning.

TEST AGAINST NUMERICAL SOLUTIONS

The numerical scheme of the finite difference method, which utilizes an implicit scheme with an explicit linearization as proposed by *Haverkamp et al.* [1977] was used by *Poulovassilis et al.* [1991] to obtain solutions for four types of soil (two sands, a soil and sand mixture, and Yolo light clay). Their numerical results appear to support their hypothesis of identical profiles. We employ the same numerical scheme to obtain solutions for Yolo light clay, which is expected to exhibit the largest difference in the two profiles among the four types of soil used by Poulovassilis et al. The same value $q/K_s = 1.637$ is considered as was used by *Poulovassilis et al.* [1991]. Time and space steps are kept constant, being equal to 0.001 hour and 1 cm, respectively.

Our results differ from those of Poulovassilis et al. both in relation to t_c and T and in relation to profiles. Values of t_c = 49.6 hours and T = 90.9 hours are found as opposed to t_c = 51 h and T = 86.7 h by *Poulovassilis et al.* [1991]. The two profiles from our results are shown in Figure 1, and it can be seen that there exists a clear difference between the two profiles at the wetting front, and that the R profile at $t = T$ penetrates deeper than the F profile at $t = t_c$. The ratio of the two volume changes is $\Delta W_F(t_c)/\Delta W_R(T) = 0.941$. For the higher flux ratio of $q/K_s = 50$, the divergence is even more marked and the volume ratio is even lower at 0.914.

THEORETICAL CONSIDERATIONS

The above analysis using both analytical and numerical solutions demonstrates that the hypothesis of identical profile suggested by *Poulovassilis et al.* [1991] is not exact for all conditions and for all soils. It remains to identify the error in reasoning in the "theory" section of the original paper.

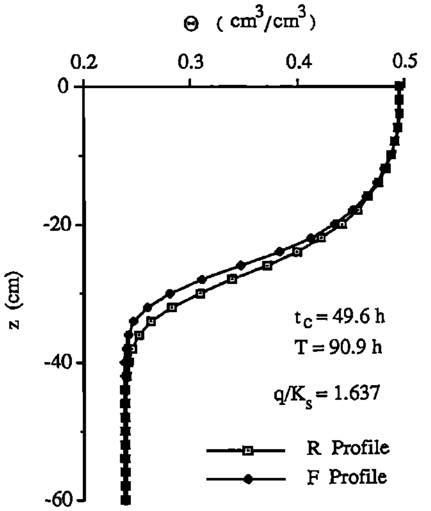

Fig. 1. A comparison of R and F profiles.

Poulovassilis et al. [1991, p. 1405] argued that for it to be possible that the R profile at $t = T$ penetrates deeper than the F profile at $t = t_c$,

> one has to accept that the velocities of descent of the relatively small water contents during the R process must be greater than those observed during the F process, which opposes the physics of infiltration. . . .

We would suggest that this argument is not conclusive since the R process with smaller velocities but acting over a longer period T could penetrate deeper than the F process with larger velocities but acting over a shorter period t_c. The analytical and numerical tests summarized above indicate that it is likely to be true in general that the R process always penetrates deeper than the F process except for a delta $D(\Theta)$ function which postulates and therefore produces identical rectangular profiles.

For most types of sandy soils, the two profiles are not very different as $D(\Theta)$ is usually close to a delta function, i.e., it is small when the soil is relatively dry and rapidly increases at near saturation. However, the proposition is not true in general and may be seriously in error for heavy soils.

CONCLUSIONS

1. The hypothesis of *Poulovassilis et al.* [1991] that the profiles at the instant of ponding with a flux at the surface equal to a constant are identical for the two cases of constant flux and instantaneous ponding boundary conditions is a good approximation but not exactly true for all conditions.
2. The degree of approximation is best for sandy soils and worst for heavy soils.
3. The approximation would appear to be accurate enough for many practical computations of soil moisture accounting for most soils and conditions.

REFERENCES

Clothier, B. E., J. H. Knight, and I. White, Burger's equation: Application to field constant-flux infiltration, *Soil Sci.*, *132*, 255–261, 1981.

Fujita, H., The exact pattern of concentration dependent diffusion in a semi-infinite medium, II, *Text. Res. J.*, *22*, 823–827, 1952.

Haverkamp, R., M. Vauclin, J. Touma, P. S. Wierenga, and G. Vachaud, A comparison of numerical simulation models for one-dimensional infiltration, *Soil Sci. Soc. Am. J.*, *41*, 285–294, 1977.

Knight, J. H., and J. R. Philip, Exact solutions in nonlinear diffusion, *J. Eng. Math.*, *8*, 219–227, 1974.

Kühnel, V., Scale problems in soil moisture flow, Ph.D. thesis, Natl. Univ. of Ireland, Dublin, 1989.

Kühnel, V., G. C. Sander, J. C. I. Dooge, and J. P. J. O'Kane, Comparative study of simple computer models of soil moisture flow, *Proj. CLI-038-EIR(H)*, *Rep. 5*, Dep. of Civ. Eng., Univ. College Dublin, 1986.

Kühnel, V., J. C. I. Dooge, G. C. Sander, and J. P. J. O'Kane, Duration of atmosphere-controlled and of soil-controlled phases of infiltration for constant rainfall at a soil surface, *Ann. Geophys.*, *8*(1), 11–20, 1990.

Philip, J. R., Recent progress in the solution of nonlinear diffusion equation, *Soil Sci.*, *117*, 257–264, 1974.

Poulovassilis, A., P. Kerkides, S. Elmaloglou, and I. Argyrokastritis, An investigation of the relationship between ponded and constant flux rainfall infiltration, *Water Resour. Res.*, *27*(7), 1403–1409, 1991.

Sander, G. C., V. Kühnel, T. Brandyk, J. C. I. Dooge, and J. P. J. O'Kane, Analytical solutions to the soil moisture flow equations, *Eur. Econ. Community/Natl. Bd. for Sci. and Technol. Proj. CLI-038-EIR(H)*, *Rep. 3*, Dep. of Civ. Eng., Univ. College Dublin, 1986.

Wang, Q. J., and J. C. I. Dooge, Limiting cases of land surface fluxes, *J. Hydrol.*, in press, 1993.

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