

Scaling effects on moisture fluxes at unvegetated land surfaces

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Abstract. As part of a larger study on spatial variability of land surface processes, the authors explore the sensitivity of land surface modules for climate models to the method of simulating the unsaturated subsurface flows. By examining the behavior of a number of different subsurface modules, it is shown that the surface fluxes, and consequently the water balance throughout the year, vary widely for different simulations of subsurface conditions. Typical results are presented for a specified climates and soil types. In order to reduce the complexity and computation time for the subsequent sensitivity studies, it is shown that a linearized module displays the range of behavior expected in practice. For given forcing functions of precipitation and potential evaporation, varying the depth of the modelled soil layer and changing the lower boundary conditions greatly influence the annual values of the components of the water balance. Monte Carlo simulations are used to demonstrate that spatial variation in soil properties produces large variation in runoff and compensating variations in deep drainage with a much smaller variation in evaporation. Finally, it is shown that for a given coefficient of variation in soil scaling properties, the effect on the effective large-scale sorptivity is insensitive to the type of statistical distribution used to describe the variation.

1. Introduction

This paper is concerned with the effects of the choice of soil moisture modelling assumptions and of the spatial variability of soil properties on elements of the water balance at both point and catchment scales and, in particular, on the moisture fluxes of infiltration and evaporation at the land surface in climate models. The soil moisture is the prime factor in switching the control of surface fluxes from the soil to the atmosphere and vice versa [Wang and Dooge, 1994], and consequently the modelling of soil moisture and its spatial variation is critical for water balance calculations. It has long been known that the variability of soil properties at point and fields scales is considerable, even for a supposedly uniform soil [Beckett and Webster, 1971; Nielsen and Bouma, 1985], but the effect at catchment scale remains a topic for investigation.

From the point of view of the hydrologist, the key test of any macroscale model is its ability to predict the amount and distribution of actual evapotranspiration. Such objective comparisons of climate models which do exist show wide variations between their predictions even for standard conditions. The following strategy is proposed for dealing with this problem: (1) Identify the land surface modules in climate models which are able to simulate the greatest range of variation in actual evapotranspiration found in these objective comparisons; (2) identify the most sensitive and least sensitive parameters of these modules; and (3) examine the sensitivity of the most significant parameters to spatial variability.

The approach used here is to simplify the problem by finding the most significant factors and neglecting the variables and parameters of lesser sensitivity. The main objective is to assess the broad effect of modelling assumptions and of soil variability on the macroscale water balance and to suggest simple

methods for estimating effective macroscale parameters for soil properties of hydrologic significance.

2. Context of the Present Study

The study reported here was undertaken as part of a research project on spatial variability of land surface processes (SLAPS) under the EPOCH Programme of the European Union. The project was aimed at bridging the gap between climate models that simulate land surface processes at a general circulation model (GCM) scale (10^5 m) and hydrologic models that simulate the same processes in more detail at a local scale (down to 1 m). The details of the individual GCMs, the methodology of the main intercomparison studies, and of the individual supplementary studies are described in the final project report [Dooge *et al.*, 1993] and summarized in a paper presented to the European Union Copenhagen Symposium on Climate Change and Impacts [Dooge, 1993].

An important feature of the project was a carefully planned objective comparison of the one-dimensional versions of the nine numerical models involved under prescribed standard surface and soil conditions and standard forcing inputs from the U. K. Meteorological Office single-column model [Dooge, 1993, p. 646]. The comparison covered a range of conditions, including two different climate types (humid and semiarid), two types of soils (Castelnu loam and Lubbon sand), and two types of land cover (bare soil and pasture). These were tested in all eight possible combinations over a simulation period of 1 year, with a 1-year run-in for initializing the soil moisture levels and an output time step of 30 min.

In all cases the annual estimates for actual evaporation differed widely between the models. For example, Table 1 shows the estimated actual evaporation given by seven models for the test year for bare soil and a humid climate [Dooge, 1993, pp. 656–657]. The precipitation for these models was approximately 668 mm, and the potential evapotranspiration was 419 mm for loam and 424 mm for sand. It is clear that the differ-

Table 1. Comparison of Different Modelling Approaches for Bare Soil and Humid Climate

Model	Actual Evaporation, mm	
	Loam	Sand
Paris	157	154
Dublin	240	227
Bracknell	246	240
Bologna	251	271
M.I.T.	314	314
Toulouse	359	366
Hamburg	386	413

ences between the individual models (157–386 mm for loam and 154–413 mm for sand) is far greater than the difference between the individual results for loam and sand (–13 to 27 mm). Clearly, further study is urgently required to understand and reduce these large differences. The corresponding comparison for pasture in a humid climate showed a much lower range of estimates of actual evapotranspiration, that is, from 392 to 477 mm.

Among the supplementary studies undertaken at the Centre for Water Resources Research in University College Dublin was an examination of the effect of the choice of module for simulating the unsaturated soil zone on the elements of the water balance. The results of this study for the loam soil are described below.

2.1. Variety in Soil Modules

The key problem in linking climate models to hydrologic models is the reduction of the potential evaporation predicted by the climate model to a reliable estimate of actual evaporation through the agency of the hydrologic model. The key element in this reduction process is the simulation of the moisture distribution and movement in the zone of unsaturated soil by means of a soil module that is sufficiently realistic but still simple enough to be coupled interactively with the climate model. The bare soil moisture modules used in the basic intercomparison of the SLAPS project exhibited a wide variety of features with respect to (1) depth of soil column (from 0.1 to 1.0 m), (2) number of soil layers (from 1 to 10), and (3) type of lower boundary condition, for example, zero flow, free drainage, or an empirical relationship [Dooge *et al.*, 1993; Dooge, 1993]. Table 2 shows the various combinations of features in the individual models used in the basic intercomparison for a bare soil surface [Dooge, 1993].

This first step in the study was an investigation as to whether the variations in depth of soil and nature of lower boundary condition in Table 2 would be sufficient to account for the range of actual evaporation estimates shown in Table 1. The initial test was made for the most easily computable model, that is, the linearized Richards' equation for a single column of soil.

2.2. Results for the Linearized Equations

A flexible, one-dimensional, linearized, formulation of the Richards' equation for unsaturated flow in a porous medium [Philip, 1966] was used and has the double advantage of providing analytical solutions which give some preliminary insights and also ensures a stable and very rapid computation for the numerical simulations [Wang and Dooge, 1994]. The basic Richard's equation can be written as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[D(\theta) \frac{\partial \theta}{\partial z} \right] + \frac{\partial K(\theta)}{\partial z} \quad (1)$$

where θ is moisture content, t is time, z is vertical distance measured downwards from the surface, $K(\theta)$ is hydraulic conductivity, and $D(\theta)$ is diffusivity, defined by

$$D(\theta) = K(\theta) \frac{\partial}{\partial \theta} [h(\theta)] \quad (2)$$

where $h(\theta)$ is matric potential.

Equation (1) can be linearized by taking the diffusivity as a constant and assuming a linear relationship between hydraulic conductivity and moisture content:

$$K(\theta) = a\theta + b \quad (3)$$

An analytical solution can be derived in terms of the error function and complementary error function. The approach can be easily extended to a quasi-linear solution which greatly facilitates solution by numerical methods. Comparison of its solution with other closed form solutions (both linear and nonlinear) indicates its usefulness as a tool for preliminary analysis [Philip, 1969; Wang, 1992; Wang and Dooge, 1994] since it gives a good approximation of overall fluxes even though the local moisture distribution in the vertical may not conform to that found in real soils.

Three different lower boundary conditions for the soil column were investigated: (1) no flow, (2) free drainage, and (3) a saturated soil (i.e., approximating a water table). The analytical solution of the linearized equation indicates that the memory length of the soil module is directly proportional to the depth of the soil column for the free drainage case. This was confirmed by numerical simulation which showed that for certain soils, the "usual" 1 year running-in period for the model was inadequate to establish appropriate starting conditions for the intercomparison. All subsequent results reported here are thus based on a 5-year run-in period to avoid any problems with initial conditions.

The numerical model was used to explore the effect of the depth of the soil column and the lower boundary condition used in the different models (see Table 2) on the elements of the water balance for a loam soil in a humid climate. The results, shown in Table 3, show a variation in actual evaporation greater than the variation between the different models of Table 2. It can be seen from Table 3 that for the loam soil, a depth of 1.0 m or more is sufficient to eliminate dependence on depth for the free drainage case and a depth of 1.5 m is sufficient for the no-flow condition. Since there is no evidence (as yet) for preferring any particular model from Table 2 over the others, the initial studies on the effect of spatial variability

Table 2. Features of the Bare Soil Modules Used in SLAPS 2

Model	Depth, m	Soil Layers	Drainage at Lower Boundary
Bologna	1.00	1	empirical
Bracknell	0.10	1	free drainage
Cambridge	0.10	1	none
Dublin	1.00	10	free drainage
Hamburg	1.00	1	empirical
Paris	1.00	2	none
Toulouse	0.10	2	none

were carried out using the linearized model with its substantial computational advantages.

3. Effect of Spatial Variability in Soil Properties

Having verified that the linearized Richards' equation could simulate the range of variation in actual evaporation, the same simplified model was adapted for studying scaling effects. Even for such a simplified model there are two physical properties (conductivity and diffusivity) where variation has to be taken into account.

The analysis of the spatial variability of soil properties and the upscaling of water fluxes from microscale to macroscale is greatly assisted by the concept of soil similarity introduced by Miller and Miller [1955, 1956]. This is based on the hypothesis that the bulk of the variation in soil properties is due to geometrical scaling and that soils of a given type at neighboring locations differ only in some characteristic length scale (λ). The basis of the approach is outlined in Appendix A together with the resulting scaling, in terms of the characteristic length λ , of the local soil properties (soil moisture potential (h), hydraulic conductivity (K), and hydraulic diffusivity (D)) and also the field scale parameters (sorption (S) and ultimate rate of infiltration (K_0)) for the linearized Richard's equation.

Accepting that the linearized Richards' equation can span the variations in actual evaporation revealed by the SLAPS intercomparison, it is used as the basis of a first approach to linking point scales to catchment scales. This is first done by using the one-dimensional single-column solution to explore the effect on the overall water balance of variation of parameters between members of an assemblage of single soil columns. The limitations of the one-dimensional approach are then examined by contrasting this case of noninteracting vertical columns, which corresponds to zero lateral conductivity, with the case of infinite lateral conductivity, which can also be postulated as a one-dimensional problem and solved in terms of moisture potential [Dooge, 1995]. Finally, the lack of sensitivity to the exact form of the spatial variability is demonstrated.

Here, the Miller and Miller scaling factor (λ) is used as the basis for a Monte Carlo numerical experiment investigating the effect on elements of the long-term water balance of spatial variability of 1000 independent soil columns of specified depth. A separate value of the scaling factor, λ , was generated for each soil column using a random number generator. This value was used to scale the hydraulic conductivity/moisture content relationship and the diffusivity by multiplying the base values for that soil by λ^2 and λ , respectively, as indicated by Appendix

Table 3. Effect of Depth of Soil Column and Boundary Condition on Actual Evaporation for Loam Soil With Humid Climate

Depth of Soil Column, m	Actual Evaporation, mm	
	Free Drainage	No Flow
0.1	195	269
0.3	229	345
0.5	237	369
0.8	239	370
1.0	240	372
1.5	240	375
2.0	240	375

Table 4. Effect of Spatial Variability in Soil Hydraulic Properties for Loam Soil With Humid Climate

Water Quantity, mm/yr	Original Results Without Any Variability	Original Results With Variability in Soil Properties		
		CV = 0.1	CV = 0.5	CV = 1.0
Actual evaporation	240	241	242	224
Runoff	128	130	153	205
Drainage	301	298	273	239

A (equations (A5) and (A6)). Each soil column is simulated independently using these soil properties and the same forcing inputs of precipitation and potential evaporation. Performance statistics are gathered from the ensemble of simulations. The distribution of λ was for convenience taken as the discrete form of the gamma distribution, that is, an Erlang distribution, which is particularly simple to generate. It is shown in the next section that the average field-scale properties are not sensitive to the type of distribution assumed.

For the case of the loam soil the introduction of the spatial variability of λ only changes the annual values of the water balance equation by 3 mm or less for a coefficient of variation (CV) of 0.1 (see Table 4). For higher values of CV the major changes occur in the balance between runoff and drainage, with evaporation not changing by as much (Table 4). The resulting distribution of the annual runoff total is positively skewed while that of deep drainage is negatively skewed.

4. Effect of Type of Distribution

The usual approach to dealing with spatial variability of soil properties is to estimate fluxes for each of an assemblage of vertical columns, each with a different value of λ , and then to average these over the area of interest. For any theoretical solution of Richard's equation the effect of the spatial variability on each parameter of the infiltration equation can be expressed as the ratio of the expected value of the parameter to the value based on the mean value of the basic soil parameter λ over the same area. In the present study the basic variation is taken as a gamma distribution and the soil module is taken as the simplest case of constant hydraulic conductivity and constant diffusivity [Dooge, 1995]. For this simple case the infiltration is given by

$$f(t) = \frac{S}{2(t)^{1/2} + K_0} \quad (4)$$

where K_0 is the rate of ultimate infiltration and S is the sorptivity given by

$$S = 2(\theta_{\text{sat}} - \theta_0) \left(\frac{D}{\pi} \right)^{1/2} \quad (5)$$

The ratio of the expected value of the sorptivity over the whole ensemble, $E[S(\lambda)]$, to the sorptivity value based on the expected value of λ is given by

$$\frac{E[S(\lambda)]}{S(E[\lambda])} = \frac{\Gamma[CV^2 + 1.5]}{\Gamma[CV^2]} \frac{1}{CV^3} \quad (6)$$

where $\Gamma[]$ is the gamma function. Values of this ratio are given in the third column of Table 5. This scaling ratio can be

Table 5. Effect of Distribution on Field-Scale Parameters, Sorptivity

CV (λ)	Value of $E[S(\lambda)]/S[E(\lambda)]$			
	$K_h = 0$, Uniform	$K_h = 0$, Gamma	$K_h = 0$, Lognormal	$K_h = \infty$, All Distributions
0.0	1.000	1.000	1.000	1.000
0.1	1.004	1.004	1.004	1.005
0.2	1.015	1.015	1.014	1.020
0.3	1.034	1.033	1.033	1.044
0.4	1.061	1.058	1.057	1.077
0.5	1.098	1.090	1.087	1.118
0.6	...	1.128	1.122	1.166
0.8	...	1.220	1.264	1.281
1.0	...	1.329	1.297	1.414

shown to be relatively insensitive to the statistical distribution used. For instance the scaling ratio for a uniform distribution of λ is shown in the second column of Table 5. For the log-normal distribution we have

$$\frac{E[S(\lambda)]}{S[E(\lambda)]} = (1 + CV)^{3/8} \quad (7)$$

which is shown in the fourth column of Table 5. Since field data for soil spatial variability indicate a unimodal distribution close to the gamma distribution, the sensitivity to type of distribution can be ignored in the early stages of model development.

5. Effect of Horizontal Conductivity

These numerical experiments based on independent vertical columns are open to the serious objection that the neglect of horizontal flows makes the approach quite unrealistic. The sensitivity of the results to horizontal flow (i.e., between columns) is briefly summarized in this section. The results of previous sections are based on the assumption of zero horizontal conductivity, that is, no flow between columns. For the contrasting case of infinite horizontal conductivity, no gradient of moisture potential can be sustained, and it can be shown that the flux problem for the assemblage of columns of varying λ can be postulated and solved as a one-dimensional problem in moisture potential involving constant conductivity and constant specific water capacity [Dooge, 1995]. The moisture content at a given depth will vary from column to column and the initial condition needs careful handling. However, the result is simple since it can be shown that

$$\frac{E[S(\lambda)]}{S[E(\lambda)]} = [1 + CV^2(\lambda)]^{1/2} \quad (8)$$

and it is thus independent of the distribution used. The results for some given values of CV are shown in the fifth column of Table 5. Equation (8) is thus an exact scaling factor for sorptivity in the case of infinite horizontal conductivity.

For both limiting cases the scaling factor for the ultimate rate of infiltration, K_0 , in equation (4) is given exactly by

$$\frac{E[K_0(\lambda)]}{K_0(E[\lambda])} = (1 + CV^2)^{1/2} \quad (9)$$

A comparison between columns three and five of Table 5 suggests that a good approximation for upscaling is obtained by

applying the scaling given by (9) to both terms on the right-hand side of (4).

6. Conclusions

The current wide variation in the estimates of moisture fluxes at the land surface in linked atmosphere land climate models needs to be reduced in order to improve the usefulness of such models. This paper has shown the following for a linearized soil module:

1. The variation in the parameters of a linearized soil module, between limits customarily used, produces variations in actual evaporation of comparable magnitude to the variation between the values estimated by different models used in the objective comparison test of the SLAPS program (Tables 1 and 3).
2. For shallow soils the surface flux estimates are very sensitive to the choice of lower boundary condition for the soil column (Table 3).
3. For all conditions tested the no-flow boundary condition, which is closest to the bucket-type assumptions, did not exhibit the full range of possible variation in evaporation (Table 3).
4. Actual evaporation is not as sensitive to spatial variability in soil properties as either surface runoff or drainage. The main effect of variability is in the splitting of the water balance between surface runoff and drainage (Table 4).
5. The effective field-scale sorptivity is only slightly sensitive to the type of distribution used to characterize the Miller and Miller scale parameter (λ) and to the degree of horizontal moisture movement (Table 5). Consequently, for all cases we can accept (9) as a close upper bound on an upscaling factor for the field sorptivity.
6. The effect of the type of probability distribution chosen for the Miller and Miller [1956] scale parameter (λ) does not effect the ultimate rate of infiltration at field scale, which depends only on the mean value and the variance of λ (equation (9)). Combined with the previous conclusion, this means that the total infiltration as defined by (4) can be upscaled by (9).
7. Taken together, these results indicate that the sensitivities of water balance simulation to various forms of spatial variability within the grid scale (Tables 3, 4, and 5) are 1 or 2 orders of magnitude less than the variations between standard climate models under identical forcing (Tables 1 and 2). This suggests that the most fruitful approach to the improvement of the hydrologic component of climate models would probably lie in carefully designed intercomparison tests in which the soil moisture module is standardized.

Appendix A: Miller and Miller Scaling of Soil Hydraulic Properties

Water is held in the unsaturated zone because of capillary forces arising from the curvature of the air-water interface in accordance with the basic relationship at the particle scale [Childs, 1969]

$$p_a - p_w = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \quad (A1)$$

where p_a is the pressure in the soil air, p_w is the pressure in the soil water, σ is the surface tension of water, and r_1 and r_2 are

the principal radii of curvature at the air-water interface. For two or more geometrically similar soils with identical shape and packing of particles and with identical contact angles, the volumetric moisture content (θ) can be related to matric potential (h) by the general expression

$$\theta = F_1\left(\frac{\lambda}{\sigma} \gamma h\right) \quad (\text{A2})$$

where λ is the Miller and Miller characteristic length, σ is the surface tension parameter, γ is the weight density of the fluid, and the functional $F_1(\)$ is the same for each of the similar soils. By combining the Navier-Stokes equation at the pore scale and the Darcy equation at the continuum scale of the representative elementary volume, we can write the relationship between conductivity (K) and volumetric moisture content (θ) as

$$\frac{\mu K}{\gamma \lambda^2} = F_2(\theta) \quad (\text{A3})$$

where μ is the dynamic viscosity of the fluid, γ is the weight density of the fluid, and the functional F_2 is the same for geometrically similar soils.

From the Miller and Miller scaling result of (A2) we can deduce that for a given moisture content, we have the scaling rules

$$h \propto \lambda^{-1} \quad (\text{A4})$$

and from the scaling result of (A3),

$$K \propto \lambda^2 \quad (\text{A5})$$

for matric potential (h) and hydraulic conductivity (K), respectively. By using these in the definition for hydraulic diffusivity, we get

$$D \propto \lambda^2 \lambda^{-1} = \lambda \quad (\text{A6})$$

which is the variation of diffusivity with geometrical scale.

If we now consider an assemblage of (say) 1000 independent soil columns, each with its individual value of scaling factor, λ , giving different but constant values for K and D , then the rate of infiltration for each column is given by (4) as

$$f(t, \lambda) = \frac{S(\lambda)}{2(t)^{1/2} + K_0(\lambda)} \quad (\text{A7})$$

and for the assembly of columns

$$E[f(t, \lambda)] = \frac{E[S(\lambda)]}{2(t)^{1/2} + E[K_0(\lambda)]} \quad (\text{A8})$$

where $E[\]$ is the mathematical expected value operator, calculated by integrating the product of the relevant quantity and its probability distribution function. The problem is to write the terms on the right-hand side of (A8) in terms of the parameters of the statistical distribution of λ . Since in this case both D and K are assumed constant, the defining equation (2) for D can be integrated to give

$$D(\lambda)(\theta_{\text{sat}} - \theta_0) = -K(\lambda)h_0 \quad (\text{A9})$$

where h_0 is the initial value of the matric potential which for equilibrium is constant over all columns, θ_0 is the initial moisture content, and θ_{sat} is the moisture content at saturation. Since $D \propto \lambda$ and $K \propto \lambda^2$, the condition for an equilibrium initial moisture distribution among the columns must be

$$(\theta_{\text{sat}} - \theta_0) \propto \lambda \quad (\text{A10})$$

Substituting from (A6) and (A10) into (5) gives

$$S \propto \lambda^{3/2} \quad (\text{A11})$$

as the scaling rule for the sorptivity.

The ultimate rate of infiltration (K_0) scales as hydraulic conductivity, that is,

$$K_0 \propto \lambda^2 \quad (\text{A12})$$

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