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Hydrodynamics of the capture zone of a partially penetrating well in a confined aquifer

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Abstract. In the pump and treat approach to the problem of managing a contaminated aquifer, a key problem is to design an effective capture system that collects only the polluted groundwater without allowing any of it to escape. At present, it is customary to design a capture system using fully penetrating withdrawal wells. Very often, however, only part of the vertical thickness of the aquifer is contaminated, so the question may arise whether a more efficient capture system can be achieved using partially penetrating wells. Very little work has been done on the application of partially penetrating wells to this problem. A new semianalytic method that can be used in determining the geometry of the capture zone for steady state flow to a partially penetrating well that is screened from the top (or from the bottom) of a confined aquifer has been developed. By combining the velocity potentials for flow to the well with that for the regional flow field, a three-dimensional velocity potential that can be used in determining the complete geometry of the capture surface has been developed. The results have shown that with a constant pumping rate the maximum horizontal extent of the capture surface at the top (or bottom) of the aquifer increases as the degree of penetration decreases. As one would expect, the maximum vertical extent increases as the depth of penetration increases. Thus, if one knows the actual location of the contaminant plume, an appropriate combination of the degree of penetration and pumping rate can be selected to create an effective capture zone.

Introduction

A common method of managing an aquifer that has become contaminated with undesirable chemicals is to extract the polluted groundwater and reduce the concentration of the contaminants to an acceptable level. The treated water is either reinjected into the aquifer or released at the surface. The first problem, of course, is to locate the horizontal and vertical extent of the contaminant plume and the level of concentrations. After the plume has been defined, a critical problem in controlling the chemical transport so that no further pollution can occur is to design an effective capture system. This means providing a system of wells so that withdrawal of contaminated groundwater from one or more wells will effectively stop any further migration of the pollutants. A recent book by *Gorelick et al.* [1993] provides an excellent review of these subjects.

The capture surface can be considered an imaginary surface that divides the fluid particles moving to the well from the rest of the groundwater that bypasses the withdrawal well. This surface can also be called a dividing stream surface. Contaminant particles on the inside of the capture surface move to the withdrawal well, whereas those on the outside of this surface do not. The movement of particular particles along the capture surface toward the stagnation point is subject to bifurcation: they may or may not be captured by the withdrawal well. In designing an effective capture system, one needs to investigate the magnitude, direction, and influence of the regional hydraulic gradient. Then, one has to measure the transmissivity and establish the nature of the boundary conditions for the contaminated aquifer. A final step is to determine the number and locations of the withdrawal wells, the degree of penetration, and the pumping rates.

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A number of workers have developed analytical solutions to define the capture zone that can be produced by wells that penetrate the entire thickness of a confined aquifer [*Aravin and Numerov*, 1965; *Goldberg*, 1976; *Polubarinova-Kochina*, 1977; *Bochever et al.*, 1979; *Javandel and Tsang*, 1986; *Luckner and Shestakov*, 1991; *Grubb*, 1993]. For two-dimensional flow, *Javandel and Tsang* [1986] introduced the idea of capture zone–type curves. *Domenico and Schwartz* [1990] have described this approach in designing either injection or withdrawal systems. *Gorelick et al.* [1993] provide a detailed procedure for optimizing capture and containment systems.

Very often, there may be instances when only part of the vertical thickness of the confined aquifer has been contaminated, and the question may arise whether a more efficient capture system can be achieved using partially penetrating wells. This kind of system leads to a problem of three-dimensional flow. To our knowledge, very little work has been done on this problem in developing a capture zone for contaminant plumes. The limiting case of three-dimensional, axisymmetric flow to a well with zero penetration in an aquifer of infinite thickness was developed by *Milne-Thomson* [1960], *Yuan* [1967] and *Goldberg* [1976]. *Haitjema and Kraemer* [1988] proposed another type of analytical approach for modeling

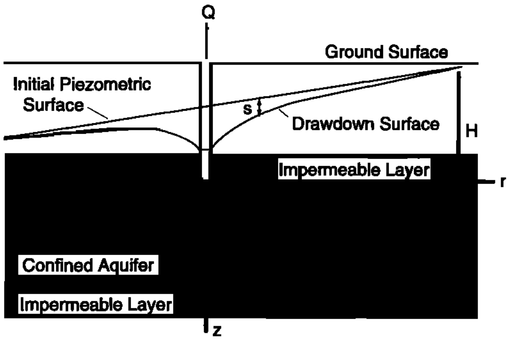

Figure 1. Sketch showing a partially penetrating well in a confined aquifer with regional groundwater flow.

partially penetrating wells in a confined aquifer, and they analyzed an example of flow to a partially penetrating well in a confined aquifer with a circular recharge area at the upper boundary.

Harmsen et al. [1991] have published a numerical solution using a particle-tracking model for flow to a well that is partially penetrating in an unconfined aquifer. *Tiedeman and Gorelick* [1993] have recently presented a new three-dimensional groundwater management model for a shallow unconfined aquifer. In the case of partial penetration, their numerical simulations indicated that flow to a given well was insensitive to the location of the well screen interval. Despite the development of various numerical three-dimensional models that can be used to determine the effects of partially penetrating wells on the capture zone, analytic and semianalytic methods provide very useful tools for this problem from both the theoretical and practical viewpoints.

Before one can determine the number and location of withdrawal wells that will be used in establishing a capture system for a given site, the basic solution for flow to a single well is needed. Therefore the purpose of this paper is to develop a method that can be used to define the geometrical configuration of the three-dimensional surface which separates the groundwater that will be captured by the withdrawal well from the rest of the regional flow system. This method may then be used in developing multiwell extraction systems.

Two approaches can be used in determining the stream surface. One of them is based on the definition of the stream function [*Yih*, 1957; *Matanga*, 1993]. The other is based on determining the coordinates of the family of streamlines that are on the stream surface. In this paper we shall use the second approach. We start with an analytical expression describing the velocity potential distribution around a partially penetrating well in the presence of a uniform flow field. This is followed by deriving a set of ordinary differential equations describing the three-dimensional streamlines around the well. The coordinates of the stagnation point for the flow system are then obtained. Starting from the close neighborhood of the stagnation point, the trajectories of a series of streamlines that are on the capture surface can finally be determined. The last step is carried out using a numerical technique that incorporates the Runge-Kutta approximation method.

Statement of Problem

In setting up this problem for steady state flow, it is assumed that the confined aquifer is horizontal and of large radial extent. It is also assumed that the aquifer is homogeneous, isotropic, and uniform in thickness. It is further assumed that the regional uniform groundwater flow, with a Darcy velocity U, is in the direction of the negative x axis. Dispersion and diffusion are considered to be negligible relative to advection. A partially penetrating well located at the origin of the coordinate system extracts contaminated water at rate Q. The well is assumed to have an infinitesimal radius and to be screened from the top of the aquifer; skin effects are neglected. (The method developed here will, of course, be the same if the well is screened from the bottom.) Figure 1 shows a sketch of these conditions. We are interested in finding the capture zone of the well.

The partially penetrating well, acting in an aquifer where the regional hydraulic gradient is uniform, produces a three-dimensional flow field in a certain region around the well. Because the vertical well is orthogonal to the horizontal regional flow field, flow around the partially penetrating well is not axisymmetrical. As a result, the problem becomes very complex.

Solution of Problem

Steady state axisymmetric flow to a single well can be described by the well-known equation

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial z^2} = 0 \qquad (1)$$

where s is the drawdown and r and z are cylindrical coordinates. For the particular case of a partially penetrating well that is screened from the top of the aquifer down to a depth l, the boundary conditions are

$$\frac{\partial s}{\partial z} = 0 \qquad z = 0 \quad \text{and} \quad z = b, \qquad (2)$$

$$s = 0 \qquad r = R \qquad (3)$$

$$\lim_{r\to 0}\left(r\frac{\partial s}{\partial r}\right) = -\frac{Q}{2\pi K l} \qquad 0 < z < l \qquad (4)$$

$$\lim_{r\to 0}\left(r\frac{\partial s}{\partial r}\right) = 0 \qquad l < z < b \qquad (5)$$

where R is the radius of influence of the well and K and l are the hydraulic conductivity and the depth of penetration of the well, respectively. Using the method of finite cosine transforms, *Javandel* [1982] developed the following solution for (1) subject to boundary conditions (2)–(5):

$$s(r, z) = \frac{Q}{2\pi K b}\left[\ln\left[\frac{R}{r}\right] + \sum_{n=1}^{\infty}\frac{2b}{n\pi l}\sin\left[\frac{n\pi l}{b}\right]\cos\left[\frac{n\pi z}{b}\right] \cdot \left\{K_0\left[\frac{n\pi r}{b}\right] - I_0\left[\frac{n\pi r}{b}\right]\frac{K_0\left[\frac{n\pi R}{b}\right]}{I_0\left[\frac{n\pi R}{b}\right]}\right\}\right] \qquad (6)$$

where I_0 and K_0 are zero-order modified Bessel functions of the first and second kind, respectively.

An examination of (6) reveals that for $R/b > 1.5$, the term involving I_0 can be neglected. Therefore, (6) can be simplified to

$$s(r, z) = \frac{Q}{2\pi Kb}\left[\ln\left[\frac{R}{r}\right] + \sum_{n=1}^{\infty}\frac{2b}{n\pi l}\sin\left[\frac{n\pi l}{b}\right]\cos\left[\frac{n\pi z}{b}\right]K_0\left[\frac{n\pi r}{b}\right]\right] \quad (7)$$

In Cartesian coordinates, (7) can be written as

$$s(x, y, z) = \frac{Q}{2\pi Kb}\left[\ln\left[\frac{R}{(x^2+y^2)^{1/2}}\right] + \sum_{n=1}^{\infty}\frac{2b}{n\pi l} \cdot \sin\left[\frac{n\pi l}{b}\right]\cos\left[\frac{n\pi z}{b}\right]K_0\left[n\pi\frac{(x^2+y^2)^{1/2}}{b}\right]\right] \quad (8)$$

Others, such as *Hantush* [1957, 1961] and *Verigin et al.* [1977], have solved the problem for a partially penetrating well for both steady state and transient conditions with arbitrary locations for the screened interval using the method of superposition. For the particular case when the well is screened down from the top, their solutions reduce to the expression given in (7).

Three-Dimensional Velocity Potential

Applying the principle of superposition, the velocity potential, ϕ, for the combined flow field can be expressed by

$$\phi = \phi_w + \phi_u \quad (9)$$

where ϕ_w is the velocity potential due to withdrawal by the well and ϕ_u is the velocity potential due the regional flow. The common expression for the velocity potential is

$$\phi = -Kh + C \quad (10)$$

where C is an arbitrary constant. For flow toward a well, (10) takes the form

$$\phi_w = -K(H_0 - s) + C_w \quad (11)$$

where H_0 is the hydraulic head in the aquifer in the absence of any drawdown and C_w is a constant.

For uniform regional flow of groundwater parallel to and in the direction of the negative x axis, the velocity potential has been given [*Milne-Thomson*, 1960] as

$$\phi_u = -Ux + C_u \quad (12)$$

where U is the Darcy velocity and C_u is a constant for the uniform regional flow. Therefore the combined velocity potential may be given as

$$\phi = \frac{Q}{2\pi b}\left\{\ln\frac{R}{(x^2+y^2)^{1/2}} + \frac{2b}{\pi l}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left[\frac{n\pi l}{b}\right] \cdot \cos\left[\frac{n\pi z}{b}\right]K_0\left[n\pi\frac{(x^2+y^2)^{1/2}}{b}\right]\right\} - KH_0 - Ux + C \quad (13)$$

where $C = C_w + C_u$.

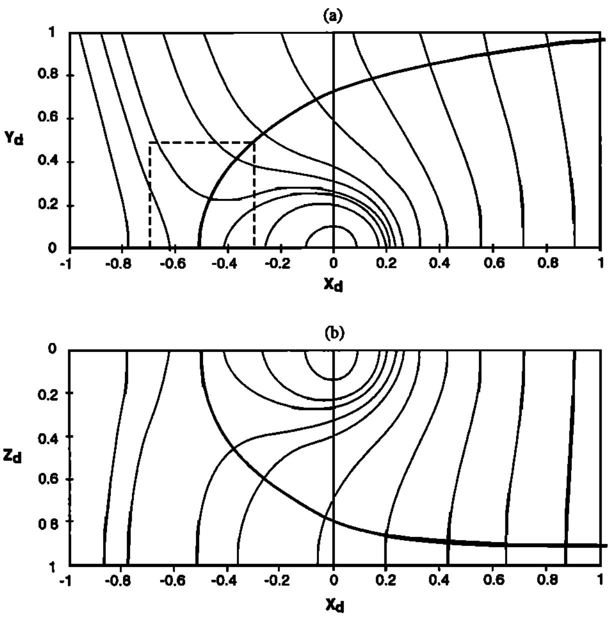

Figure 2. Velocity potential distribution and curves of the maximum extent of the capture zone (a) on the plane $z_d = 0$ and (b) on the plane $y_d = 0$, for the case of $Q = 5 \times 10^{-4}$ m^3/s, $K = 6.1 \times 10^{-5}$ m/s, $b = 45.7$ m, $l = 4.57$ m, and a regional hydraulic gradient of 0.0025 (dimensionless parameters: $Q_d = 1.58$, and $l_d = 0.1$). The dashed line shows the area that is expanded in Figure 3.

Although the combined flow field is not axisymmetric, it should be noted that the resulting field is irrotational, because both (11) and (12) satisfy the Laplace equation.

The velocity potential function given by (13) can be used to generate equipotential surfaces anywhere within the flow field. To examine the nature of these equipotential surfaces, it is convenient to introduce the following dimensionless parameters. The dimensionless coordinates will be defined by

$$x_d = \frac{x}{b} \qquad y_d = \frac{y}{b} \qquad z_d = \frac{z}{b} \quad (14)$$

The dimensionless degree of penetration will be given as

$$l_d = \frac{l}{b} \quad (15)$$

and the dimensionless flow rate by

$$Q_d = \frac{Q}{b^2U} \quad (16)$$

An example of the nature of these equipotential surfaces in the vicinity of the withdrawal well where $l_d = 0.1$ is shown by the profiles in Figure 2. Figure 2a presents equipotentials on the horizontal plane at the top of the aquifer, where $z_d = 0$, and shows clearly how the flow equipotentials are changed in the vicinity of the extraction well. Streamlines are orthogonal to the equipotential lines, and on this particular plane the critical streamline that separates flow to the well from the regional flow that bypasses the well is shown by the heavy line. Figure 2b shows the equipotential configuration on the vertical plane along the x axis. This figure also shows the effect of the extraction well on the equipotentials close to the well. Once again, the critical streamline that separates flow to the well from the regional flow that bypasses the well is shown by the heavy line.

The two critical streamlines shown in Figure 2 represent the maximum extent of that portion of the flow field that is captured by the withdrawal well. In other words, both of these streamlines are on the capture surface, and of course they must converge at the stagnation point. The process for locating these critical streamlines, as well as the stagnation point, will be described below.

Three-Dimensional Capture Surface

Having developed an expression for the three-dimensional velocity potential, given by (13), the next step is to derive a set of ordinary differential equations that will enable one to find those streamlines that lie on the capture surface in three-dimensional space. By definition, a streamline is a line such that the tangent at any point is the direction of the velocity vector at that point. The streamlines are therefore the solution to the following set of the ordinary differential equations:

$$\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w} \tag{17}$$

where u, v, and w are components of the velocity vector and can be determined by obtaining partial derivatives of the overall velocity potential expression given by (13). Using dimensionless parameters, one obtains

$$u_d=\frac{-Q_d}{2\pi}\frac{x_d}{x_d^2+y_d^2}\left[1+\frac{2(x_d^2+y_d^2)^{1/2}}{l_d}\sum_{n=1}^{\infty}\sin(n\pi l_d)\cdot\cos(n\pi z_d)K_1[n\pi(x_d^2+y_d^2)^{1/2}]\right]-1 \tag{18}$$

$$v_d=\frac{-Q_d}{2\pi}\frac{y_d}{x_d^2+y_d^2}\left[1+\frac{2(x_d^2+y_d^2)^{1/2}}{l_d}\sum_{n=1}^{\infty}\sin(n\pi l_d)\cdot\cos(n\pi z_d)K_1[n\pi(x_d^2+y_d^2)^{1/2}]\right] \tag{19}$$

$$w_d=\frac{-Q_d}{\pi l_d}\left[\sum_{n=1}^{\infty}\sin[n\pi l_d]\sin[n\pi z_d]K_0[n\pi(x_d^2+y_d^2)^{1/2}]\right] \tag{20}$$

where $u_d = u/U$, $v_d = v/U$, and $w_d = w/U$.

To obtain a solution of (17), it must be decomposed into two equations, and by substituting for u_d, v_d, and w_d from (18) through (20), one obtains

$$\frac{dy_d}{dx_d}=\frac{v_d}{u_d}=f(x_d, y_d, z_d) \tag{21}$$

$$\frac{dz_d}{dx_d}=\frac{w_d}{u_d}=g(x_d, y_d, z_d) \tag{22}$$

where $f(x_d, y_d, z_d)$ and $g(x_d, y_d, z_d)$ are two functions defined as

$$f(x_d, y_d, z_d)=\frac{y_d}{x_d}\left[1-\frac{1}{1+(Q_dx_d/2\pi)[(1+P_1)/(x_d^2+y_d^2)]}\right] \tag{23}$$

$$g(x_d, y_d, z_d)=\frac{(Q_dP_2)/(\pi l_d)}{1+(Q_dx_d/2\pi)[(1+P_1)/(x_d^2+y_d^2)]} \tag{24}$$

and P_1 and P_2 are given below:

$$P_1=\frac{2(x_d^2+y_d^2)^{1/2}}{l_d}\sum_{n=1}^{\infty}\sin(n\pi l_d)\cos(n\pi z_d)\cdot K_1(n\pi(x_d^2+y_d^2)^{1/2}) \tag{25}$$

$$P_2=\sum_{n=1}^{\infty}\sin(n\pi l_d)\sin(n\pi z_d)K_0(n\pi(x_d^2+y_d^2)^{1/2}) \tag{26}$$

Simultaneous integration of (21) and (22), subject to a given initial condition, gives the coordinates of an arbitrary streamline in the flow system under consideration.

The stagnation point in a flow system is defined as a point where all components of the velocity vector are 0. This point is on the capture surface and is the specific location where all streamlines on the capture surface converge. To find the coordinates of the stagnation point $(x_{d,\mathrm{st}}, y_{d,\mathrm{st}}, z_{d,\mathrm{st}})$, the components of the velocity vector are set equal to 0 in (18)–(20), and this yields

$$x_{d,\mathrm{st}}=\frac{-Q_d}{2\pi}\left\{1+2\frac{|x_{d,\mathrm{st}}|}{l_d}\sum_{n=1}^{\infty}\sin(n\pi l_d)K_1(n\pi|x_{d,\mathrm{st}}|)\right\} \tag{27}$$

$$y_{d,\mathrm{st}}=0$$

$$z_{d,\mathrm{st}}=0$$

Note that the x coordinate of the stagnation point in (27) is given in an implicit form. It can be seen that the coordinates of the stagnation point are a function of only two parameters, Q_d and l_d.

For the limiting case where $l_d = 1.0$, the solution to the system of (21)–(26) becomes

$$y_d=\pm\frac{Q_d}{2}-\frac{Q_d}{2\pi}\tan^{-1}\left(\frac{y_d}{x_d}\right) \tag{28}$$

which describes the capture surface for the fully penetrating well as a function of only one parameter, Q_d [*Javandel and Tsang*, 1986]. For this case, (27) simplifies to

$$x_{d,\mathrm{st}}=-\frac{Q_d}{2\pi} \tag{29}$$

In the case of zero penetrating wells in a semi-infinite aquifer one may use the following expression to obtain the coordinate of the stagnation point [*Milne-Thomson*, 1960]

$$x_{\mathrm{st}}=-\left(\frac{Q}{2\pi U}\right)^{1/2} \tag{30}$$

Having determined the location of the stagnation point, the next problem is to locate a sufficient number of streamlines so that the geometry of the capture surface can be adequately defined. As discussed above, an initial condition is needed in order to be able to simultaneously solve (21) and (22). Therefore, to locate streamlines that are on the capture surface, one has to select as an initial condition coordinates of points that are also on this surface.

However, if an initial point is selected that is close enough to the stagnation point so as to be on, or very near, the capture surface, its coordinates can be used in the solution process. Through experience, it was found that if the incremental dimensionless distance of such points from the stagnation point is in the range from 0.001 to 0.01, the result will be a streamline that is on the capture surface.

If we want to represent the capture surface by n streamlines, (21) and (22) should be solved n times, and each time with a different initial point, because away from the stagnation point, only one streamline can pass through a single point in three-dimensional space. The problem then becomes that of selecting appropriate initial values to be used in integrating (21) and (22).

An examination of the system of equations (21) and (22) reveals that due to the form of functions f and g, obtaining a general closed-form solution appears to be impossible. Therefore, a numerical technique must be used for the integration process, and a fourth-order Runge-Kutta procedure [*Ince*, 1926] was applied in solving (21) and (22). For an increment Δx_d along the x_d axis, the increments of k and m along the y_d and z_d axes can be calculated from the following equations

$$k_1 = \Delta x_d f(x_{d0}, y_{d0}, z_{d0}) \quad (31a)$$

$$m_1 = \Delta x_d g(x_{d0}, y_{d0}, z_{d0}) \quad (31b)$$

$$k_2 = \Delta x_d f(x_{d0} + \tfrac{1}{2}\Delta x_d, y_{d0} + \tfrac{1}{2}k_1, z_{d0} + \tfrac{1}{2}m_1) \quad (31c)$$

$$m_2 = \Delta x_d g(x_{d0} + \tfrac{1}{2}\Delta x_d, y_{d0} + \tfrac{1}{2}k_1, z_{d0} + \tfrac{1}{2}m_1) \quad (31d)$$

$$k_3 = \Delta x_d f(x_{d0} + \tfrac{1}{2}\Delta x_d, y_{d0} + \tfrac{1}{2}k_2, z_{d0} + \tfrac{1}{2}m_2) \quad (31e)$$

$$m_3 = \Delta x_d g(x_{d0} + \tfrac{1}{2}\Delta x_d, y_{d0} + \tfrac{1}{2}k_2, z_{d0} + \tfrac{1}{2}m_2) \quad (31f)$$

$$k_4 = \Delta x_d f(x_{d0} + \Delta x_d, y_{d0} + k_3, z_{d0} + m_3) \quad (31g)$$

$$m_4 = \Delta x_d g(x_{d0} + \Delta x_d, y_{d0} + k_3, z_{d0} + m_3) \quad (31h)$$

$$k = \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (31i)$$

$$m = \tfrac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4) \quad (31j)$$

The general procedure for defining the geometry of the capture surface for a given problem involves the following steps. (1) Determine the coordinate of the stagnation point along the x axis using (27). (2) Determine the maximum horizontal extent of the capture surface on the horizontal plane, $z_d = 0$, by integrating equation (21). (3) Determine the maximum vertical extent of the capture surface on the vertical plane, $y_d = 0$, by integrating equation (22). (4) Using results for the maximum extent from steps 2 and 3, construct an arbitrary profile for the capture surface on a $y_d - z_d$ plane (i.e., a plane that is orthogonal to the regional flow field) at a small incremental distance ($\Delta x_d < 0.01$) upgradient from the stagnation point. The shape of the resulting curve will be a semiellipse that becomes more circular as the well penetration decreases. (5) Select about five equally spaced points on this arbitrary profile that can be used as initial points in calculating the coordinates for the streamlines that pass through each of these points. From the results, it will be obvious where additional points should be selected to reveal the details of the geometry of the capture surface. Experience has shown that 10 to 12 streamlines usually provide sufficient data to define the capture surface. In carrying out the numerical integration using the Runge-Kutta method, it was found that up to 30 summation terms in (25) and (26) were needed to establish a reliable result.

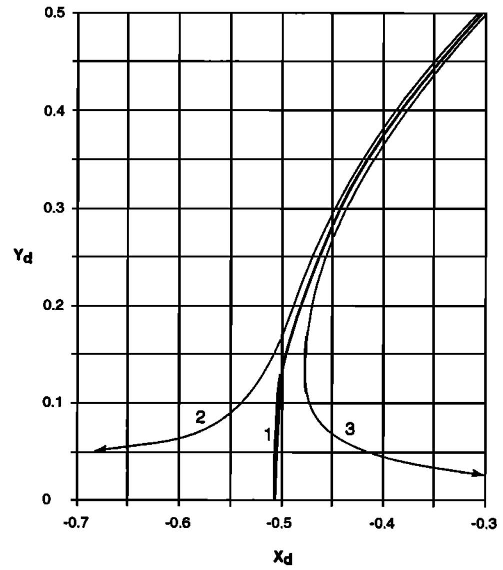

Figure 3. Expanded plot of streamlines on the plane $z_d = 0$ in the near vicinity of the stagnation point showing (line 1) dividing streamline on the capture surface, (line 2) streamline starting at $x_d = 2.9$ with $\Delta y_d = 0.001$ on the outside of the dividing streamline, and (line 3) streamline starting at $x_d = 2.9$ with $\Delta y_d = 0.001$ on the inside of the dividing streamline.

Discussion of Results

Accuracy of Calculated Coordinates

One of the questions that will obviously be raised for the semianalytic procedure that has been developed in this work is the accuracy with which one can calculate the coordinates of the streamlines that lie on the capture surface. To investigate this problem, two points were chosen just off an extension of the streamline shown in Figure 2a at $x_d = 2.9$, which is relatively far from the location of the stagnation point on this horizontal surface. One point was chosen at $\Delta y_d = 0.001$ above the streamline, which places the point just outside the capture surface. The other point was chosen at the same dimensionless distance below this streamline, which places the point just inside the capture surface. The streamlines that pass through these two points were then mapped by calculating their coordinates using (21).

The expanded view in Figure 3 shows how the two streamlines diverge from the particular streamline (labeled as 1) that is on the capture surface and essentially converges on the stagnation point. As they come close to the stagnation point, the streamline (labeled as 2) that originated outside the capture surface remains outside, and the streamline (labeled as 3) that originated inside the capture surface converges on the withdrawal well. In effect, unless one examines the locations of streamlines in the near vicinity of the stagnation point, the deviations from the capture surface are so small that they

Table 1. Coordinates of Stagnation Point Along the x_d Axis

l_d	$Q_d = 0.1$	$Q_d = 1$	$Q_d = 2$	$Q_d = 4$	$Q_d = 6$	$Q_d = 8$	$Q_d = 10$
0.01	−0.126	−0.403	−0.579	−0.856	−1.111	−1.371	−1.647
0.05	−0.121	−0.401	−0.578	−0.856	−1.111	−1.371	−1.647
0.1	−0.108	−0.396	−0.574	−0.854	−1.109	−1.370	−1.646
0.2	−0.075	−0.378	−0.562	−0.845	−1.103	−1.366	−1.644
0.3	−0.052	−0.351	−0.541	−0.831	−1.093	−1.360	−1.640
0.4	−0.039	−0.317	−0.514	−0.812	−1.080	−1.351	−1.635
0.5	−0.032	−0.281	−0.481	−0.789	−1.064	−1.340	−1.628
0.6	−0.027	−0.248	−0.446	−0.761	−1.044	−1.328	−1.621
0.7	−0.023	−0.220	−0.411	−0.731	−1.023	−1.314	−1.614
0.8	−0.020	−0.196	−0.377	−0.700	−1.000	−1.300	−1.606
0.9	−0.017	−0.176	−0.346	−0.668	−0.977	−1.286	−1.598
1	−0.016	−0.159	−0.318	−0.637	−0.955	−1.273	−1.592

cannot be seen on a plot such as that in Figure 2. The same procedure was used to examine the streamline on the vertical surface in Figure 2b, with the same result. It appears that the accuracy of solutions derived from (21) and (22) is quite adequate for practical purposes.

Locating the Stagnation Point

The location of the stagnation point can be determined by using (27). Table 1 gives the coordinates of the stagnation points for values of Q_d from 0.1 to 10 and values of l_d ranging from 0.01 to 1.0. In the downgradient direction from the well, the stagnation point lies further and further from the well, either as a result of decreasing penetration, l_d, or as a result of increasing values of Q_d. As can be seen in Table 1, as Q_d increases, the effect of changes in l_d on the location of the stagnation point becomes less and less. It can also be seen that for a given value of Q_d, the location of the stagnation point is not changed significantly as l_d becomes less than 0.1.

The entries in Table 1 for small values of Q_d and l_d have been checked against the numerical values of (30), which give the coordinate of the stagnation point for a zero penetrating well in a semi-infinite aquifer. The comparison is quite satisfactory.

Maximum Horizontal Extent of Capture Surface

The maximum horizontal extent of the capture zone at the top of the aquifer where $z_d = 0$ was computed from (21). For the particular case where $Q_d = 1$, the curves in Figure 4a show how the horizontal extent of the capture zone increases as the penetration of the withdrawal well decreases. This increase in the spread of the capture zone is a result of the increasing flow velocity into the well as the degree of penetration decreases for a fixed total flow rate Q. These results indicate that in field situations where the contaminant plume is in the uppermost layers of the aquifer, the use of partially penetrating wells should be more effective and more economical than the use of completely penetrating wells.

It should be noted in Figure 4a that as the distance from the withdrawal well increases in the upgradient direction, each curve tends toward an asymptotic value of y_d. It is also of interest to note that when $l_d < 0.1$, the maximum extent of the capture zone approaches the result for a point sink/source, i.e., a well of zero penetration.

In examining the effect of withdrawal rate, we have found that when Q_d exceeds about 10, the horizontal extent of the capture zone approaches that for a fully penetrating well. This is because when Q_d increases, the stagnation point moves further from the well and $|x_{d,\mathrm{st}}|$ becomes more than 1 (see Table 1). In this case, $(x_d^2 + y_d^2)^{1/2}$ exceeds 1, and the Bessel functions, K_1 and K_0, vanish. Hence the terms that take into account the effects of the partially penetrating well, P_1 (equation (25)) and P_2 (equation (26)), also vanish. As a result, regardless of the value of l_d, the solution of (21) and (22) approaches that of (28).

Maximum Vertical Extent of Capture Zone

The maximum vertical extent of the capture zone is computed from (22) and will of course be along the x axis at depths that depend on the magnitude of Q_d and the degree of penetration. For the particular case where $Q_d = 1$, the curves in Figure 4b show how the depth of the capture surface increases as the degree of penetration increases. It can be seen that when $x_d > 1$, each of these curves is at an asymptotic value. Another important consideration is the effectiveness of a partially penetrating well in capturing contaminants that are below the bottom of the open interval in the well. Reference to the curve for $l_d = 0.1$ in Figure 4b shows a capture zone that extends below $z_d = 0.6$. This implies that all streamlines within this

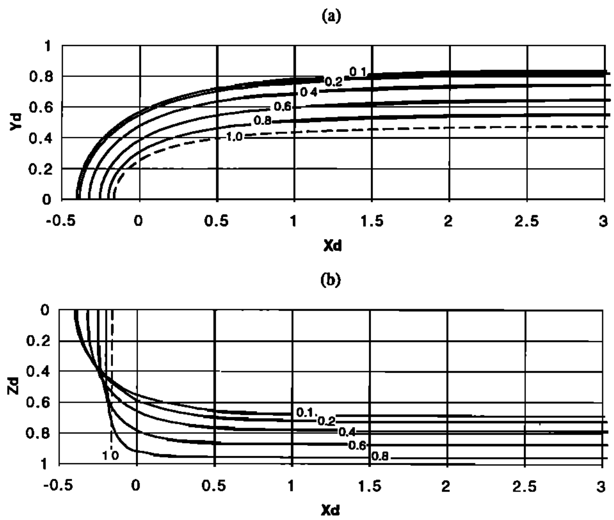

Figure 4. Curves of (a) the maximum horizontal extent of the capture zone along the plane $z_d = 0$ and (b) the maximum vertical extent along the plane $y_d = 0$, for the case of $Q_d = 1.0$. Numbers on the curves are the values of the degree of penetration, l_d.

capture zone along this vertical section must converge on a well that penetrates only one-tenth the thickness of the aquifer.

For this same case of one-tenth penetration, Figure 5 shows the effect of variations in Q_d from 0.5 to 5.0. It can be seen that as the withdrawal rate exceeds $Q_d = 2$, the capture zone along the x axis is practically at the bottom of the aquifer.

The effectiveness of partial penetration in capturing contaminants can also be demonstrated by constructing cross sections that are orthogonal to those shown in Figure 4, i.e., along vertical planes where x_d is constant. For example, Figure 6a shows three profiles of the capture surface as x_d increases from -0.3 to 3.0 for the case of $Q_d = 1.0$. As x_d exceeds 3.0, the profile is not changed appreciably. It is important to compare the profile for $l_d = 0.1$ with that of the fully penetrating case where $l_d = 1.0$. This is shown in Figure 6b, and the shaded area demonstrates the significantly greater region in the upper half of the aquifer that is captured by a well with one-tenth penetration.

This approach of using a semianalytic method to determine the geometry of the capture surface for flow to a partially penetrating well in a confined aquifer can be extended to treat a number of other similar problems. Some examples are (1) wells that are screened at one or more arbitrary intervals, (2) a system of wells with arbitrary locations, (3) a system of wells in an unconfined aquifer, (4) a system of wells in an aquifer with vertical anisotropy, and (5) a system of injection wells where it is necessary to confine the disposal volume within some prescribed region.

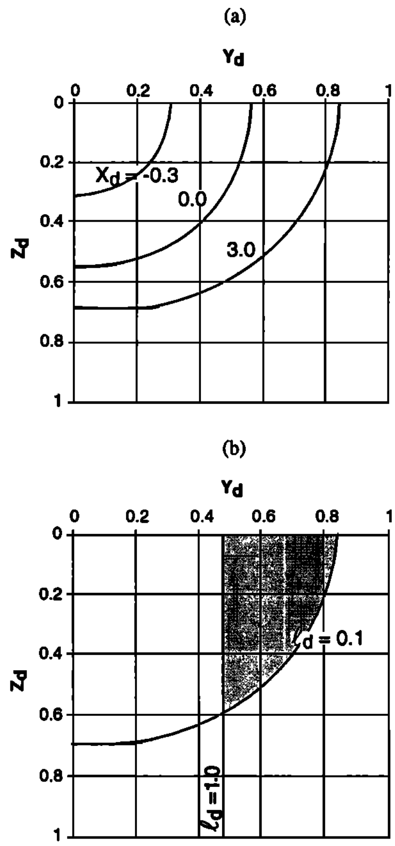

Figure 6. Curves of the maximum extent of the capture surface along the vertical planes y_d-z_d for the case of $Q_d = 1.0$ (a) for $l_d = 0.1$ at different distances along the x_d axis and (b) for $l_d = 0.1$ and $l_d = 1.0$ at $x_d = 3.0$.

Conclusions

A new semianalytic method that can be used in determining the geometry of the capture zone for steady state flow to a partially penetrating well that is screened from the top of a confined aquifer has been developed. By combining the velocity potentials for flow to the well with that for the regional flow field, a three-dimensional velocity potential, equation (13), that can be used in determining the complete geometry of the capture surface has been developed.

The first step is to locate the position of the stagnation point using (27), which was derived from the three-dimensional velocity potential. The second step is to describe the geometry of the capture surface by solving for a system of three-dimensional streamlines that converge on the stagnation point. The coordinates of the streamlines can be obtained by solving the set of ordinary differential equations in (17). This is accomplished by decomposing the equations in (17) into two ordinary differential equations, (21) and (22), that are numerically integrated simultaneously. The process of integration requires a starting point which must be located sufficiently close to the stagnation point. A particular capture surface can be determined by specifying two dimensionless parameters, Q_d and l_d.

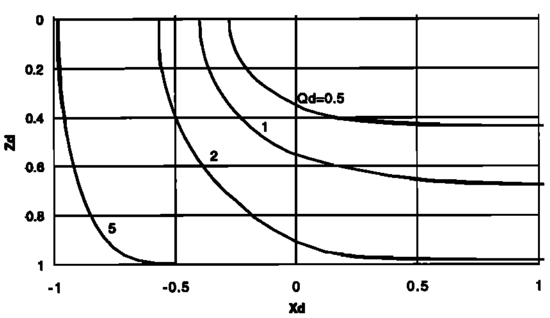

Figure 5. Curves of the maximum vertical extent of the capture zone along the plane $y_d = 0$ for $l_d = 0.1$ and values of Q_d from 0.5 to 5.0.

The results of calculation have shown that the maximum horizontal extent of the capture surface at the top (or at the bottom) of the aquifer increases as the degree of penetration decreases. As one would expect, the maximum vertical extent increases as the depth of penetration increases. Thus, if one knows the actual location of the contaminant plume, an appropriate combination of degree of penetration and pumping rate can be selected to create the most effective capture zone. In field situations where the contaminant plume is in the uppermost layers of the aquifer, the use of partially penetrating wells should be preferable to the use of completely penetrating wells.

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