ACCURACY OF AREAL RAINFALL ESTIMATES.

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Benton's rule.—A question has probably arisen in the mind of almost everyone having to do with areal rainfall estimates as to the dependability of the results. Such estimates when based on a sufficient number of good precipitation records are generally accepted as fairly reliable. The number of records available is, however, often limited, and the question naturally arises, What is the relative accuracy of two rainfall estimates for the same area, one based on a very large number and the other on a limited number of records? Rainfall estimates sufficiently accurate for one purpose may not be so for another purpose. Something more than a mere qualitative opinion as to the accuracy of areal rainfall estimates is therefore desirable.

According to Sir John Benton[1]—

The least number of rainfall stations inside the boundaries of a catchment area which will afford a reasonably safe estimate of the rainfall may be assumed to be as follows:

Square miles area.

From	0	50	100	200	350	500
To	50	100	200	350	500	750
Stations needed	1	2	3	4	5	6

Since there can be no fractional stations, at least two stations must be used in accordance with this rule for an area of 50 to 100 square miles, and so on.

Benton's table is approximately represented by the formula—

$$N=1+\frac{1}{5}\sqrt{A} \qquad (1)$$

where A is the area in square miles. This rule is evidently intended to apply in the hilly countries of England and India where rainfall often varies rapidly with change of position, but where also the rainfall records are numerous, so that the requirements of the rule can often be complied with.

In the United States, outside of certain limited areas in the East, rainfall stations are seldom more frequent than one or two to the county or about one to every 660 square miles. As many stations as are required by Benton's rule are seldom available, but it does not follow that "reasonably safe" estimates of rainfall are not obtainable in the United States, especially in regions where the topography is flat and local variation of rainfall comparatively small.

Relation of rainfall range to accuracy.—The accuracy of an estimate of areal rainfall—assuming always that the individual records used are reliable—seems to depend primarily on two things:

1. The actual range of rainfall within the area.
2. The number of records used.

The accuracy depends only indirectly on the size of the area, since in general the larger the area the larger is the range of rainfall variation within its boundaries. If, for example, the rainfall gradient is uniform along the axis of a drainage basin, then the range would vary with the length or approximately as the square root of the area. For a given number of stations the accuracy of the average is proportional to the range of variation between the amounts for the different stations.

From the laws of probability it is well known that the probable error of the mean of a number of observations varies inversely as the square root of the number. The same relation holds for the average error and other measures of precision. In other words, the probable error of an areal rainfall estimate can be expressed in the form—

$$\delta=K\frac{R}{\sqrt{N}} \qquad (2$$

where R is the range, N the number of stations, and K a constant. If the range varies as the square root of the area, then in order that estimates shall have equal accuracy for the areas A_1 and A_2,

$$\sqrt{\frac{A_1}{N_1}}=\sqrt{\frac{A_2}{N_2}}$$

or the number of stations must be directly proportional to the size of the area instead of varying with the square root. Variation of rainfall range with $\sqrt{A}$ often holds approximately for small areas of similar physiography and exposure, but for large areas the range of rainfall may increase but little with increase of area, justifying the use of a smaller number of stations per unit area as the total area increases, as in Benton's table.

In general the range can be accurately determined from the available records for any given area, and it is better to express results as to accuracy directly in terms of range rather than in terms of area and avoid the uncertainty as to the relation of range to area. Formula (2) applies only in case the rainfall stations are not separately weighted or in case they are given equal weights, as where a straight arithmetic mean is taken. The range is determined by taking the greatest difference in mean annual rainfall at any two points within the area.

Derwent River Basin records.—The rainfall record for the basin of the River Derwent in England[2] affords an opportunity to test the agreement of experience with theory in this matter and to determine the numerical coefficient K in formula (2). Forty-two rain gages were maintained within an area of 48.87 square miles. Most of the records are complete for the 13 years 1900 to 1912, inclusive. In a few cases the record for 1900 has been supplied by interpolation using the formula—

$$P=\frac{P_a}{P_m}P_1.$$

Where P =required precipitation for 1900 at a given station.

P_1 =mean precipitation at the same station for the other years.

P_a =mean of the precipitation for 1900 at all stations having records.

P_m =mean precipitation at all stations.

Figure 1 shows the topography of the area, the locations of the rain gages, the 13-year mean for each, and the isohyetal lines. This is a rugged hilly region well dissected by streams, with precipitation generally increasing proceeding upstream.

[1] Buckley Irrigation Pocketbook, 3rd ed., 1920, p. 324.

[2] Sandeman, Proc. Inst. C. E., vol. 194, 1912-13. (H D, 7-1-22.)

Average error of arithmetic means.—In view of the large number of stations within the Derwent Basin, the true average rainfall was assumed to equal the arithmetic mean of all the records. This was checked by the application of the Thiessen method and also by planimeter measurement of the areas between contours. The errors which would have resulted if the average rainfall had been determined by taking the arithmetic means of different numbers of stations ranging from 1 to 20 was obtained by random selection of sets of the required numbers of stations. The stations were chosen by drawing their numbers from a box, the only restriction being that all stations drawn should not be adjacent. This was done for the reason that if, as an example, there were only two stations within the area and these were adjacent, they would properly be treated as in effect equivalent to a single station. In such cases the results of a second drawing were substituted. A complete drawing of each set was made independently in each case except for sets or groups of 14 to 20 stations. For these groups, to avoid excessive labor, cumulative drawing was used—that is, a station selected at random was added to each group to give the next higher group. To provide a check on the results, two series of drawings were made for all groups. The average departures of the computed rainfall as determined from a given number of stations from the true mean are summarized in Table No. 1. The results are shown graphically on Figure 2 in conjunction with a curve computed from the formula—

$$\delta = \frac{14.7\ R}{\sqrt{N}} \tag{3}$$

where δ is the average departure of the mean as determined from N stations from the true mean, expressed as a percentage of the range R.

TABLE 1.—*Average departure of mean rainfall determined by different numbers of stations from true mean, Derwent Basin, Eng.*

Number of stations used.	Number of sets drawn in each series.	Average departure from true mean.		Average of both series.	Per cent of range in mean annual rainfall.
		First series.	Second series.		
(1)	(2)	(3)	(4)	(5)	(6)
1	42	....	....	4.19	15.6
2	30	2.601	2.108	2.35	8.75
3	30	2.388	2.290	2.34	8.71
4	30	2.572	1.934	2.25	8.38
5	30	1.938	1.745	1.84	6.85
6	30	1.953	1.356	1.65	6.14
7	30	1.922	1.286	1.60	5.96
8	30	1.319	1.213	1.27	4.73
9	30	1.574	1.006	1.29	4.81
10	30	1.399	1.108	1.25	4.65
12	[1]15–[2]30	1.296	1.065	1.18	4.40
14	[1]15–[2]30	1.214	.952	1.08	4.02
16	[1]15–[2]30	.877	.985	.93	3.44
18	[1]15–[2]30	.807	.855	.83	3.09
20	[1]15–[2]30	.788	.800	.79	2.94

[1] Series 1. [2] Series 2.

Mean rainfall=46.50 inches. Range in mean=26.86 inches.

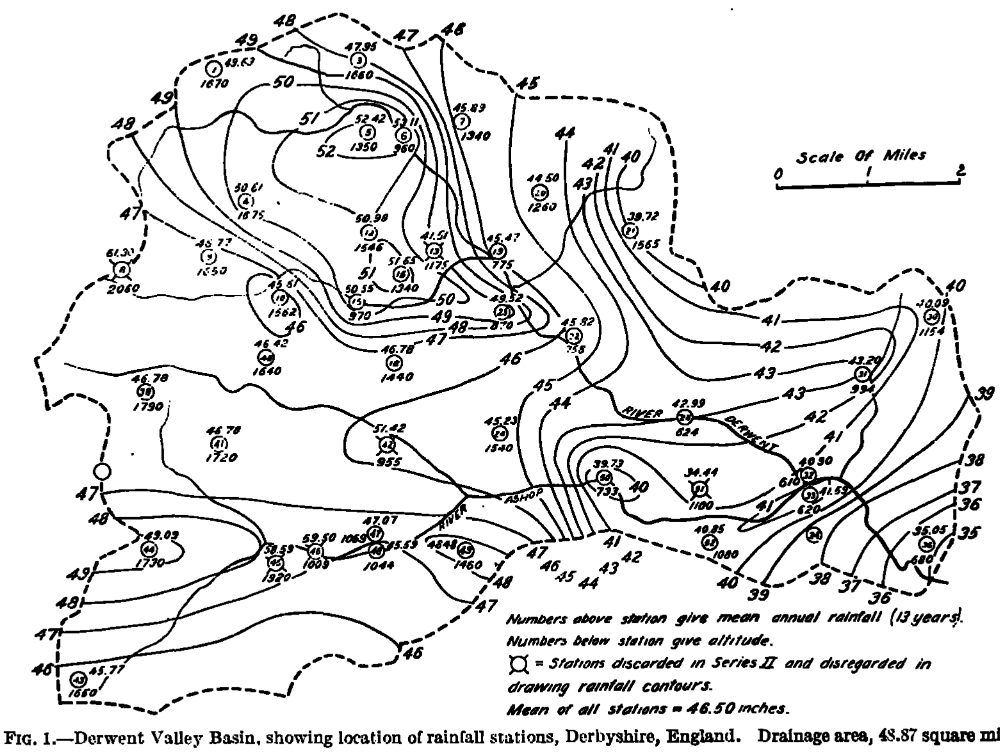

FIG. 1.—Derwent Valley Basin, showing location of rainfall stations, Derbyshire, England. Drainage area, 48.87 square miles.

Since there were only 42 stations, the plotted points on Figure 2, if extended, would show a departure approaching zero for $N=42$, whereas actually the departure should approach zero as a limit as N approaches infinity. Points for large values of N used in this study lie below the curve of Figure 2, as they should. Aside from this the plotted points faithfully confirm the applicability of the inverse square root rule to the average error of areal rainfall estimates. Expressed in terms of the true average rainfall over the area the average error of a rainfall estimate derived from the arithmetic mean of N stations is:

$$\delta = \frac{14.7\ R}{P\sqrt{N}} = \frac{0.147\ Z}{\sqrt{N}} \tag{4}$$

where R is the range or difference between the greatest and least rainfall amounts in inches within the area, P is the approximate mean areal rainfall, and Z is the range ratio in per cent of P, or

$$Z = 100 \times \frac{R}{P}. \tag{5}$$

Comparative average errors of arithmetic and Thiessen means.—The Thiessen method of estimating the rainfall over an area consists in applying to each record within or adjacent to the area a weight proportional to the percentage of that area to which the given station lies nearer than any other station.[3]

Thiessen means approaches zero as the number of stations increases. In Series IV certain stations, the records for which were apparently abnormal as compared with the several surrounding stations, were omitted. Series V was confined to the Alport Basin, a subdivision of the Derwent Basin.

Maximum departure and assured accuracy of areal rainfall estimates.—The preceding studies show the average departure, or what amounts to the same thing, the probable departure of an estimate based on a limited number of records from the true mean. This is not the same as "the probable error" of statistics, but corresponds to the "average error;" that is to say, the studies afford no assurance that the actual error of a given estimate may not be considerably greater. The user of rainfall statistics wants to be assured that the error of a given estimate does not exceed some maximum or limiting percentage of the mean rainfall.

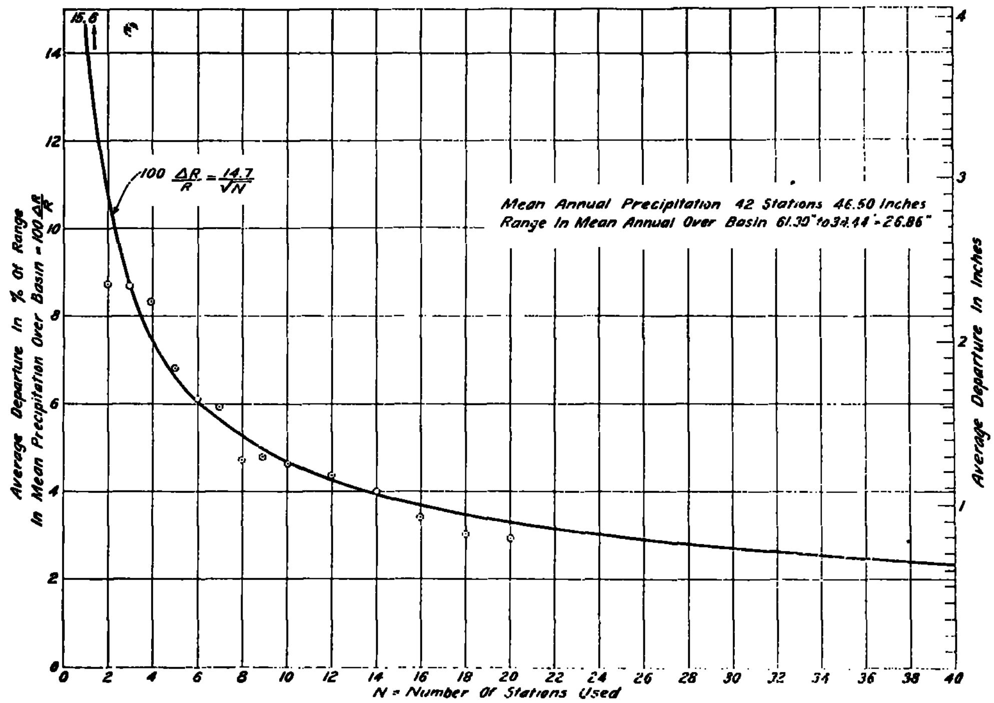

FIG. 2.—Average error in the mean annual rainfall as determined by various numbers of stations, Derwent Basin, England. Arithmetic mean.

The weights for use in the Thiessen method are determined by geometrical construction and are independent of personal equation of the computer. The theoretical advantages of this method, where available records are few in number and not uniformly distributed, are obvious.

In order to determine the accuracy of Thiessen, as compared with arithmetic, means, three series of groups of different numbers of stations were drawn and the means computed, both arithmetically and by Thiessen's method. The average errors of the two methods are shown in Table No. 2. The number of groups drawn for each number of stations is shown in column 2. In the subsequent columns the average departures are expressed in terms of inches, percentage of true mean, and percentage of range, respectively. It will be noted that in general there is a slight advantage in favor of accuracy of the true mean derived by Thiessen's method, where the number of stations available is small. This study was not extended to include groups of more than eight stations, since the difference between the arithmetic and

TABLE 2.—*Comparison of arithmetic and Thiessen mean rainfall Derwent Basin.*

SERIES III, DERWENT.—ALL STATIONS MEAN[1] 46.50 INCHES, RANGE[2] 26.86 INCHES.

Number of stations used.	Number of groups drawn.	Average departure from true mean arithmetic method.			Average departure from true mean Thiessen method.		
		Inches.	Per cent of true mean.	Per cent of range over basin.	Inches.	Per cent of true mean.	Per cent of range over basin.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	42	4.19	9.0	15.6	4.19	9.0	15.6
2	70	2.93	6.30	10.9	2.76	5.93	10.3
3	10	2.91	6.25	10.8	2.20	4.73	8.20
4	10	2.19	4.71	8.2	2.53	5.44	9.05
5	5	2.18	4.69	8.1	1.57	3.37	5.85
6	5	1.20	2.58	4.5	1.03	2.21	3.8
7	5	1.11	2.39	4.1	.85	1.82	3.2
8	5	1.65	3.55	6.1	1.32	2.84	4.9

[1] 13-year arithmetic mean of all stations used.
[2] Difference between highest and lowest 13-year record.

[3] Horton, Robert E.: Rational Study of Rainfall Data, Eng. News-Record, Aug. 2, 1917, pp. 211-213.

SERIES IV, DERWENT.—ABNORMAL STATIONS OMITTED, MEAN 45.72 INCHES, RANGE 17.37 INCHES.

Number of stations used.	Number of groups drawn.	Average departure from true mean arithmetic method.			Average departure from true mean Thiessen method.		
		Inches.	Per cent of true mean.	Per cent of range over basin.	Inches.	Per cent of true mean.	Per cent of range over basin.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	36	3.18	6.95	18.3	3.18	6.95	18.3
2	9	2.27	4.96	13.1	1.95	4.26	11.2
3	5	1.19	2.60	6.9	.86	1.79	4.7
4	3	1.91	4.17	11.0	1.58	3.45	9.1
5	2	.78	1.70	4.5	.77	1.68	4.4
6	4	.83	1.81	4.8	.48	1.05	2.9
7	2	.56	1.22	3.2	.99	2.16	5.7

SERIES V, ALPORT BASIN.—MEAN[3] 49.70 INCHES, RANGE 16.07 INCHES.

1	9	4.57	9.2	30.9	4.57	9.2	30.9
2	50	3.04	6.11	18.9	2.90	5.83	18.1
3	50	2.44	4.91	15.2	2.36	4.75	14.7
5	50	1.55	3.12	9.6	1.85	3.72	11.5
8	50	.94	1.89	5.8	1.52	3.06	9.6

[3] Average of the Thiessen and arithmetic means for the basin using all stations.

N increased, the figures probably do not show the true maximum departure in case of large numbers of stations. This is indicated by the fact that all the plotted points for more than 10 stations fall well below the envelope line AB.

In the case of Thiessen means the number of groups averaged for each value of *N* is not sufficient to determine the formula for maximum error, but is sufficient to determine the relative magnitude of the maximum errors of the two methods for different numbers of stations. The results derived from Series No. 4 are summarized in Table 4, in which the right hand columns show the difference between the maximum errors of the two methods for a given number of stations expressed as a percentage of the range. The maximum error of the Thiessen mean is smaller for groups of two to eight stations in all cases but one, and as an average the maximum error of the Thiessen mean for two to eight stations is less than that of the arithmetic mean by 5.1 per cent of the range of variation of rainfall within the area. In other words, the assured accuracy of the Thiessen mean for a limited number of stations is greater than for the arithmetic mean by about 5 per cent of the range.

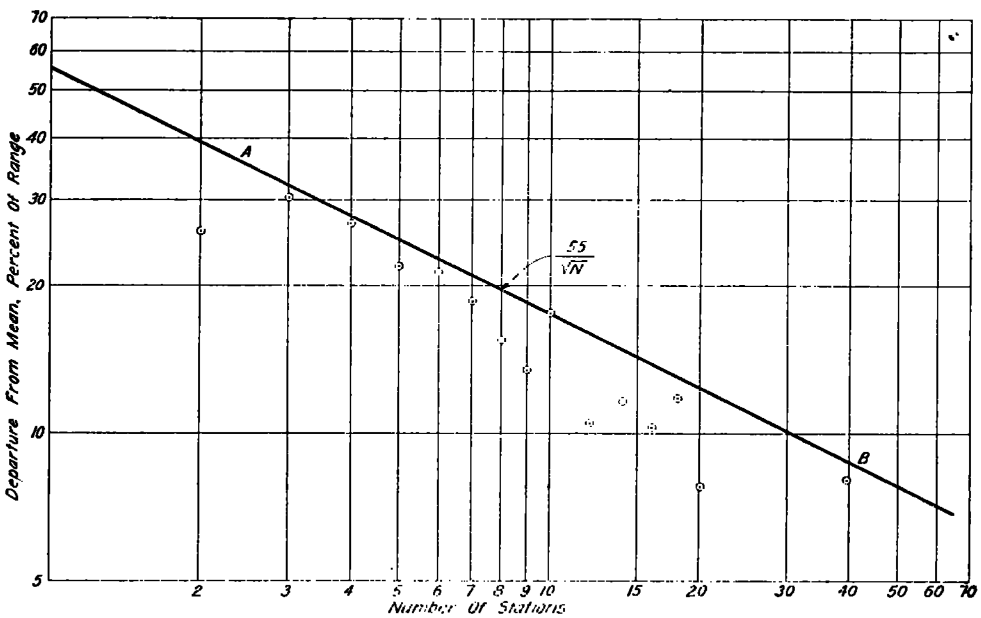

FIG. 3.—Derwent River Basin, maximum departure from true mean rainfall of estimates based on arithmetic means of different numbers of stations.

Without attempting to fix definite limits of desirable accuracy for such estimates it will serve for illustration to assume that in precise scientific investigations, as in determining water losses or for final studies of run-off, the maximum error of the rainfall estimate should, if possible, be kept within 2 per cent of the mean. For rough estimates and preliminary studies an assured accuracy of 5 per cent may be all that is required. In order to determine the greatest error which may be liable to occur in a given areal rainfall estimate, the maximum departure of the computed from the true means was determined from each set of groups of *N* stations used in the studies already described for Series 1 and 2, using arithmetic means only. The maximum departure for either series for a given number of stations and the results expressed in various forms are given in Table 3. The maximum departures expressed as percentages of the range and plotted logarithmically are shown on Figure 3. The equation of the upper envelope of the plotted points, as shown by the line AB on Figure 3, again follows the inverse square root law. Inasmuch as the number of groups of *N* stations used in this study decreased as

TABLE 3.—*Maximum departures of arithmetic mean of N stations from true mean rainfall, Derwent Basin.*

Mean annual rainfall on basin=46.50 inches.
Range in annual rainfall on basin=26.86 inches.

No. of stations N.	Maximum departure of any single group.		Larger of 2.	Per cent of range in annual rainfall.	Per cent of mean annual rainfall.
	Series I.	Series II.			
(1)	(2)	(3)	(4)	(5)	(6)
1	14.80	14.80	14.80	55.1	31.8
2	6.94	5.76	6.94	25.8	14.9
3	7.68	8.17	8.17	30.4	17.6
4	6.65	7.25	7.25	26.9	15.6
5	5.74	5.91	5.91	22.0	12.7
6	5.79	4.98	5.79	21.5	12.4
7	5.05	4.33	5.05	18.8	10.9
8	4.02	4.20	4.20	15.6	9.03
9	3.64	3.28	3.64	13.6	7.82
10	4.73	3.17	4.73	17.6	10.2
12	2.71	2.82	2.82	10.5	6.06
14	3.14	2.53	3.14	11.7	6.75
16	2.81	2.80	2.81	10.4	6.04
18	3.21	2.51	3.21	11.9	6.90
20	2.10	2.04	2.10	7.8	4.50

TABLE 4.—*Maximum departures of arithmetic and Thiessen means from true mean rainfall with different numbers of stations, Derwent Basin, England.*

Mean annual rainfall on basin=46.50 inches.
Range in annual rainfall on basin=26.86 inches.

Number of stations.	Arithmetic mean.			Thiessen mean.			Difference in per cents of range.
	Maximum departure of a single group.	Per cent of range in annual rainfall.	Per cent of mean annual rainfall.	Maximum departure of a single group.	Per cent of range in annual rainfall.	Per cent of mean annual rainfall.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	14.80	55.0	31.8	14.80	55.0	31.8	0
2	9.64	35.8	20.7	7.10	26.4	15.3	9.4
3	7.58	28.2	16.3	5.95	22.1	12.8	6.1
4	4.68	17.4	10.1	5.16	19.2	11.1	−1.8
5	3.22	12.0	6.92	3.01	11.2	6.47	0.8
6	3.37	12.5	7.25	1.55	5.78	3.33	6.7
7	3.51	13.1	7.55	1.85	6.88	3.98	6.2
8	3.06	11.4	6.58	2.06	7.66	4.44	3.7

Average of 2 to 8 stations, 4.4 per cent.

It is now possible to determine the number of records necessary in order that the estimated areal mean shall not depart from the true mean by more than an assigned amount. Expressing the limiting permissible departure Δ_g as a per cent of the mean there results—

$$N=\left(\frac{55R}{P\Delta_g}\right)^2=\frac{0.3025Z^2}{\Delta^2_g} \qquad (6)$$

or for

$$\Delta g=2\%,\ N=0.0756Z^2 \qquad (7)$$

$$\Delta g=5\%,\ N=0.0121Z^2 \qquad (8)$$

The number of stations required to provide a given assured degree of accuracy of the arithmetic mean increases as the square of the range ratio. Comparing the formulas for average and maximum departures it will be noted that the coefficient in the former is about one-fourth that in the latter formula. In other words, if the indicated maximum error of an estimate based on N records is 2 per cent, then it is probable that the actual error will be only about one-fourth this amount, or 0.5 per cent, although of course it may have any value from zero to 2 per cent. The formulas are based on records in the hilly country of England, but in view of the statistical nature of the data and the method of treatment by ratios there appears to be no reason why the formulas are not applicable elsewhere as well. This seems to follow for the reason that the statistical law of distribution of, for example, shot around the bull's eye of a target can be determined as well for a given range and rifle from observations in United States as in England.

It is to be noted that the dominant factor controlling the accuracy of areal rainfall estimates is the range ratio, or the amount of fluctuation in rainfall within the area as compared with the areal mean. It is at once evident that it may be difficult to secure a high percentage of accuracy in rainfall estimates in arid regions where the mean is small and the range ratio relatively large. In extremely wet regions greater accuracy may often be obtained, although there may be a much larger actual range, for the reason that the relative range or range ratio is usually less over an area of a given size in regions of high than in regions of low rainfall. Again, it is obviously much more difficult to secure a high degree of accuracy for estimates in mountain regions with high rainfall gradients than for areas of equal size in flat countries where the rainfall gradient is often small and comparatively uniform. Generally speaking, the rainfall range increases with the size of the area, although it may be nearly as great for a comparatively small as for a much larger area, especially in a region where the rainfall is comparatively uniform.

For the entire Mississippi River Basin the range ratio is roughly—

$$100\,\frac{60-10}{30}=167\%$$

$$\Delta_g=Z\sqrt{\frac{0.3025}{N}} \qquad (9)$$

There are at present at least 1,600 rainfall stations maintained in the Mississippi Basin, so it appears that the areal mean for this basin can be determined with an assured accuracy within 2.3 per cent and with a probability that the actual error is much less. For small areas the case is different. Throughout much of the Central States region rainfall gradients range from 1 inch in 5 to 1 inch in 10 miles over small areas, whereas the total gradient throughout a length of 100 miles may not exceed 4 to 6 inches. For purposes of comparison the range may be considered as equal to the product of the gradient by the diagonal of a square equal to the area. For an area where the range is 5 inches and the mean 30 inches 21 stations are required to provide an assured accuracy within 2 per cent, but only 3.4 stations for an assured accuracy within 5 per cent. If the range is 10 and the mean 30 inches, 53 stations are required for an assured accuracy within 2 per cent and 8 or 9 for an assured accuracy within 5 per cent. There are roundly 4,500 rainfall stations in continental United States distributed over an area of 2,970,000 square miles, or roundly 1 station to each 660 square miles. The general rule is at least one station to the county. The total number of counties is 2,741 and the actual number of stations about 1⅔ per county. The areas per station in four States are as follows:

State.	Area, square miles.	Stations.	Area.
Ohio	40,760	122	334
Michigan	57,430	123	466
New York	47,620	130	366
Massachusetts	8,040	25	321

In Massachusetts, with 1 Weather Bureau station to 321 square miles and with an average rainfall gradient of about 1 inch in 5 miles, the assured accuracy for areas of different sizes containing the usual numbers of stations is as follows:

Area (square miles).	Stations.	Range.	Range ratio.	Δ_g.
4	1	2	0.56	0.31
25	1	5	1.4	0.77
100	1	10	2.8	1.54
900	3	30	8.4	2.67
1,600	5	40	11.2	2.7
2,500	8	50	14.0	2.7
3,600	11	60	16.8	2.8
4,900	15	70	19.6	2.8
10,000	31	100	28.0	2.8

For large areas an accuracy of the arithmetic mean within 2.8 per cent is attainable under these conditions. For small areas the values of Δ_g above given are applicable only in case a rainfall station happens to fall actually within the area. The chance that this will occur is proportional to the ratio of the area to the area per station, or 321 square miles for Massachusetts. In case a station does not happen to fall within the area there is no certain method of determining the maximum departure of an estimate from the true mean except by assuming that the range is equal to the difference between the rainfall amounts at two adjacent stations, which for Massachusetts would average 18 miles, giving a range of 3.6 inches applicable to any area of less than 321 square miles.

SUMMARY.

The preceding considerations show—

1. That a high degree of reliability may be assured even where the mean rainfall over a small area is determined from a single station, provided the station falls within the area.
2. It is impracticable by the use of direct averages to obtain a high degree of certainty in the estimates of rainfall over small areas with the station spacing such as now exists in the United States unless one or more stations fall within the area.

Clearly it is also impracticable to establish and maintain a sufficient number of rainfall stations so that areal means can be determined accurately by direct arithmetic averages for all small areas. Of course the accuracy of rainfall estimates for small areas which do not contain any rainfall station may often be greatly improved by interpolation methods. In general the long-term areal means can be determined with a higher degree of accuracy than the means for individual years or months, since as a rule there are more rainfall records available in a given area than there are contemporaneous stations maintained on the area. For example, Goodenough has compiled 120 rainfall records, each of several years' duration, which have been kept at one time or another in the State of Massachusetts, an average of one record to every 67 square miles. Of course, many of these are not simultaneous, but for purposes of estimating areal means they can often be reduced to identical base periods by Fournie's or other methods.

The average error and maximum error of arithmetic areal rainfall means both vary inversely as the square root of the number of stations used.

The probable average error and maximum error or assured accuracy of an areal rainfall estimate can be determined by the use of the simple formulas summarized below.

The number of stations required to limit the error to a specified percentage of the mean varies as the square of the range ratio.

It requires only about 16 per cent as many stations to secure an assured accuracy within 5 per cent of the true mean as within 2 per cent.

The average error of an estimate is about one-fourth of the maximum error.

The arithmetic mean and Thiessen methods give identical results for a single station, and also for a very large number of stations.

For a limited number of stations, say, two to eight, the maximum error which can result from the use of the Thiessen method is less than that from the use of the arithmetic mean by an amount equal to about 5 per cent of the range.

For conditions within the United States it is possible to secure an assured accuracy of an arithmetic mean areal rainfall within 3 per cent for very large areas.

The arithmetic means for areas of 1,000 square miles or more can often be obtained with an assured accuracy within 2 per cent in the United States in regions of fairly uniform rainfall.

For very small areas an assured accuracy within 2 per cent can generally be obtained if one or more rainfall stations exist within the area.

The chance that a station exists within a given area is proportional to the ratio of the area to the average area per rainfall station.

The average area per rainfall station in the United States is 660 square miles; in the Eastern and Central States, 300 to 400 square miles.

For an area in which there is no rainfall station the assured accuracy may not be greater than that based on the assumption that the range on the area equals the total range between two adjacent outside stations.

In applying Thiessen's method the assumed assured accuracy will usually be at least equal to that indicated by the formulas given where N is the total number of stations to which Thiessen's method applies, whether within or outside the basin.

SUMMARY OF FORMULAS.

Error of arithmetic mean:	Average departure. $\delta=$	Maximum departure. $\Delta=$
Per cent range	$\frac{14.7}{\sqrt{N}}$	$\frac{55}{\sqrt{N}}$
Inches	$\frac{0.147\,R}{\sqrt{N}}$	$\frac{.55\,R}{\sqrt{N}}$
Per cent mean	$\frac{14.7\,R}{P\sqrt{N}}$	$\frac{55\,R}{P\sqrt{N}}$
Per cent mean	$\frac{0.147\,Z}{\sqrt{N}}$	$\frac{0.55\,Z}{\sqrt{N}}$

N, number of stations in area.
R, absolute range within area, inches.
P, approximate areal mean, inches.
Z, range ratio, per cent of mean$=100\,\frac{R}{P}$.