(1) The occurrence of rain wave-trains on natural ground surfaces is limited to steep slopes and high rain-intensities.

(2) The waves which form a train may have a tractive force or eroding power of the order of five times that of the same flow without waves.

(3) Standing grass inhibits the formation of rain wave-trains by preventing the instability-ratio v/Δ to attain sufficiently requisite values, but grass beaten down by rain or overland flow may accentuate wave-train erosion thereafter by releasing excess surface-detention.

(4) Wave-trains are shallow-flow phenomena and are inhibited in most natural channels by the fact that the depth is never sufficiently small to give the requisite value of v/Δ for instability.

(5) The waves in a rain wave-train apparently travel with a constant velocity $\sqrt{g\Delta}$ and remain spaced at nearly uniform intervals down the slope but increase in depth, volume, and width of base as they proceed down-slope.

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APPENDIX E--SEDDON'S AND FORCHHEIMER'S FORMULAS FOR CREST-VELOCITY OF FLOOD-WAVES SUBJECT TO CHANNEL-FRICTION CONTROL

Robert E. Horton

<u>Introduction and synopsis</u>--Among the least-understood phenomena of runoff are those involving the time of transit of the crest of a stream-rise from a given point of channel-inflow to an outlet. Practical importance attaches to this question in connection with flood-stage prediction. Time of transit is also involved where attempts are made to determine channel-outflow by the time-contour method. The validity of that method depends to a considerable extent on the accuracy with which time of transit can be determined.

There is a growing belief that the time of transit of a stream rise is governed by some type of wave-velocity and is not governed by the velocity of ordinary hydraulic flow--in other words, that the wave-crest velocity and the velocity at which water would flow at crest-depth, in accordance with, say, the Manning formula, are not the same, and that the former applies more nearly to the movement of stream-rises.

It seems obvious, although it does not appear to have been clearly pointed out before, that in natural stream-channels there is a range of conditions extending from those in which the wave-movement is almost completely subject to channel-friction control, to those in which the wave-movement is almost exclusively subject to gravity- or momentum-control, and channel-friction plays only a negligible role.

The conditions to which these two cases apply are apparently related through three factors: (1) Wave-length relative to wave-height; (2) wave-height relative to initial depth; (3) wave-height relative to hydraulic radius or, perhaps better, to the reciprocal of the hydraulic radius. Hydraulic considerations, as well as the author's experiments [see 1 of "References at end of paper], indicate clearly that the movement of channel-waves changes gradually from that subject to momentum-control to that subject to friction-control as the ratio of wave-length to wave-height increases. While apparently the type of wave-control, whether by momentum or friction, is also related to the last two factors, the nature of this relationship is as yet undetermined. As the flood-wave due to a stream-rise traverses the system of stream-channels, the crest-velocity may not only change but the laws governing the velocity may apparently change from those for momentum-control to those for friction-control or vice versa, depending on the initial depths, relative sizes and characteristics of channels, and tributary inflow for successive reaches traversed by the wave.

Complete analytical treatment of the movement of flood-waves subject to friction-control has been several times attempted without attaining results which were either simple or satisfactory. One reason is that the differential equations involved become complicated, difficult of solution, and simplifying assumptions must be made which more or less invalidate the results. No attempt will be made here to review this previous work.

There is another line of approach which involves only the use of the equation of continuity or the storage-equation in conjunction with a suitable slope-formula, such as that of Manning. This mode of approach has led to two expressions for wave-crest velocity--that of Seddon [2]

$$u = (1/w)\ (dq/dh) \tag{1}$$

and that of Forchheimer [3]

$$u = (1/w)\ [(q_2 - q_1)/(h_2 - h_1)] \tag{14}$$

The last equation has also been attributed to Boussinesq.

An expression which can readily be converted into Seddon's formula was published by Saint-Venant in 1871. Seddon appears to have been the first to put this expression directly into the form of a wave-velocity equation and to apply it to natural river flood-waves.

Forchheimer's "Hydraulics" gives an equation identical with that of Seddon and credits it to E. Lauda [9]. Seddon's first publication of his formula was in the Proceedings, American Society of Civil Engineers, December 1899.

Forchheimer does not give equation (14) in this form. Instead, he expresses q in terms of the Chezy formula for flow in a wide channel. Forchheimer's original formula applies only to such channels and is invalidated by the incorrectness of the Chezy formula. Forchheimer's formula does not appear in his treatese on "Hydraulics" [4]. Equation (14) is limited to wide rectangular channels but does not involve the Chezy formula. Equation (17)

$$u = (q_2 - q_1)/(A_2 - A_1) \tag{17}$$

is more general in form and applies to all types of channels and to all stage-discharge relationships. Neither Seddon nor Forchheimer gives equation (17) explicitly although both state the equation of continuity in a form easily reducible to this equation.

L. K. Sherman [5] gives equation (14) but refers to Seddon's paper. Harold A. Thomas [5,6] gives a derivation of equation (14). It seems clear that equation (1), giving wave-velocity in terms of the differential ratio or slope of stage-discharge relationship curves, should be designated "Seddon's formula," and equation (14), based on a similar ratio but using finite differences, may be most appropriately called "Forchheimer's formula." The meanings and limitations of these equations and the relations between them do not appear to have been generally understood. Simple derivations are given for both equations, their limitations are pointed out and it is shown that, as above given, they apply primarily to wide channels in which the hydraulic radius is sensibly equal to the depth.

Most natural stream-channels have sections which are approximate parabolas of some form. A wave-velocity equation of the same type as that of Seddon is derived in the paper for a parabolic channel-section, applicable to any channel whose form of section can be represented by a simple power-function, $w = k_w h^c$, or for which the stage-discharge relation or rating curve can be represented by a simple parabolic or power function, $q = k_q h^m$. An equation of the same type as Forchheimer's is also developed applicable to all forms of parabolic channel-sections. This may conveniently be written in the form

$$u/v_2 = [1 - (h_1/h_2)^m]/[1 - (h_1/h_2)^{(m-2/3)}] \tag{21}$$

All three equations, namely, Seddon's, Forchheimer's, and that of the author, indicate that if the initial depth is zero ($h_1 = 0$), as for a wave of the bore-type, the ratio (u/v_2), or the ratio of the wave-crest velocity to velocity of neutral flow at crest-depth, is unity. This evidently means that if uniform inflow is admitted to an initially dry stream-channel and continued long enough, a steady state will ultimately be attained at a given gage or cross-section and the velocity will be that corresponding to neutral hydraulic flow. The author's experiments on channel-waves (op. cit.) show that when water is admitted at a uniform rate to a uniform channel, initially empty, the toe of the wave travels downstream with a velocity which is related in some way, not yet determined, both to the velocity v_2 of stable flow and to $\sqrt{gd_2}$. Hence, Seddon's and Forchheimer's formulas, which give $u = v_2$, fail for this case.

In accordance with Forchheimer's formula, as the wave-height decreases relative to the initial depth h_1, the wave-velocity increases. The highest or limiting velocity as $h_1 \rightarrow h_2$ is not, however, 5/3, as indicated by the Forchheimer formula (or 3/2 if the Chezy formula is used instead of the Manning formula) but this limiting velocity varies with the form of channel cross-section, as reflected by the exponent m.

Channel-sections range from those which are triangular (m = 2) through various types of parabolic sections (m = 2 to m = 1). In natural stream-channels the form of channel-section generally changes from a nearly triangular section on headwater or first-order tributaries, to nearly the equivalent of a wide channel (m = 1) near the mouth of a large river. Consequently the wave-velocity for a given value of (h_1/h_2) also varies proceeding downstream.

A flood-wave may be considered as a mass of water having its center of gravity above the plane of equilibrium and which tends to flatten as the result of the action of gravity as it travels downstream. The tendency to flatten is resisted by channel-friction downstream from the crest but is augmented by channel-friction upstream from the crest. As a rule, the wave encounters increased depth h_1 of initial flow as it proceeds downstream. These conditions combine to increase the wave-velocity proceeding downstream. In view of the usual changes in form of channel-section and change in the ratio h_1 to h_2 as a wave travels downstream, it is not difficult to see why the simple Seddon and Forchheimer equations have not generally been found to give satisfactory results when applied to flood-wave movement in natural stream-channels.

Notation--Units are feet and seconds unless otherwise noted. Directions and velocities are positive in the direction of the initial flow. Subscripts 1 and 2 relate, respectively, to initial conditions antecedent to the wave and to conditions underneath the wave-crest.

h = stream-stage above plane of zero-discharge.
m = exponent in stage-discharge relation-formula.
n = Manning's coefficient of roughness.
p = wetted perimeter of cross-section.
q = discharge past a given section.
R = hydraulic radius of cross-section.
S = slope of water-surface.
u = velocity of wave-crest.
v = mean velocity by Manning formula for neutral flow at any given depth and slope.
k_q = coefficient in stage-discharge relation for a given reach or at a given cross-section.
k_w = coefficient in parabolic width-formula.
c = exponent in parabolic width-formula.

Seddon's formula--in 1900 James A. Seddon [2] published the formula

$$u = (1/w)\ (dq/dh) \tag{1}$$

for the velocity u or rate of downstream advance of the crest of a flood-wave. (dq/dh) is the rate of change of discharge with stream-stage. This equals the slope of the stage-discharge relation-curve (rating curve). Substantially the following proof, also another proof based on the idea of differential transfer of the water, are given in Seddon's paper. Referring to Figure 1, let ab = $d\ell$ be a short reach or element of length of stream-channel such that the wave-crest moves from a to b in the time dt. If q_2, h_2 and q_1, h_1 are average inflow- and outflow-rates and stages for this reach in the time dt, then from the storage-equation, $q_2\, dt = q_1\, dt + dS$, where $dS = [(dh_2 + dh_1)/2]\ w d\ell$ = change of storage.

The average change of stage in the reach is $dh = (dh_2 + dh_1)/2$, giving $(q_2 - q_1)\ dt = w \times dh \times d\ell$, where w = channel-surface width.

But $q_2 - q_1 = dq$, so that $dq \times dt = w\ dh\ d\ell$ and $(1/w)\ (dq/dh) = d\ell/dt$. But $(d\ell/dt)$ is a velocity, and if the crest moves from a to b in the time dt, $(d\ell/dt) = u$ = crest-velocity.

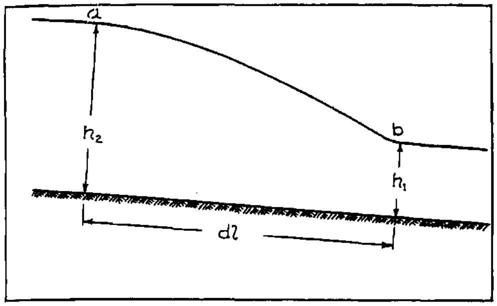


FIG.1-DERIVATION OF SEDDON'S FORMULA

From Manning's formula, for a wide rectangular channel,

$$q = w \quad (1.486/n)\ h^{5/3}\sqrt{S} = w\ k_m h^{5/3} \qquad (2)$$

where $k_m = (1.486/n)\sqrt{S}$. (3)

Also

$$(1/w)(dq/dh) = (5/3)(1.486/n)h^{2/3}\sqrt{S} = (5/3)v_2 \qquad (4)$$

where v_2 is the mean velocity of hydraulic flow at the crest-stage.

The Chezy formula leads to the result that crest-velocity does not exceed 1-1/2 times the velocity for steady flow. This result, often quoted, is incorrect. The more accurate Manning formula gives $u = (5/3)v$ for a wide channel.

Seddon formula for a rectangular channel--For a rectangular channel

$$q = k_m\ [wh/(w + 2h)]^{2/3}\ wh \qquad (5)$$

$$(1/w)\ (dq/dh) = (d/dh)\ k_m w^{2/3}\ [h^{5/3}/(w + 2h)^{2/3}]$$

$$= k_m w^{2/3}\ [(w + 2h)^{2/3}\ (5/3)h^{2/3} - (4/3)h^{5/3}/(w + 2h)^{1/3}]/(w + 2h)^{4/3}$$

$$= k_m w^{2/3}\ [5/3(w + 2h)\ h^{2/3} - (4/3)h^{5/3}]/(w + 2h)^{5/3}$$

$$= (k_m w^{2/3}/3)\ [(5p - 4h)\ h^{2/3}/p^{5/3}]$$

But $v = k_m w^{2/3}\ h^{2/3}/p^{2/3}$

$$u/v_2 = (p^{2/3}/3)\ [(5p - 4h/p^{5/3}] = (5p - 4h)/3p \qquad (6)$$

If h is small relative to w, and hence relative to p, $(u/v_2) = 5/3$ as before.

Seddon formula for a general parabolic channel--Many channels are nearly parabolic in section. For a general parabola, $w = k_w h^c$, we have

$$R = h/(c + 1),\ A = h_w/(c + 1) = k_w h^{(c + 1)}/(c + 1),\ v = k_m\ [h/(c + 1)]^{2/3} \qquad (7)$$

$$q = Av = k_m k_w\ [h^{(5/3) + c}/(c + 1)^{5/3}] \qquad (8)$$

$$(dq/dh) = k_m k_w/(c + 1)^{5/3}\ [(2/3) + c + 1]\ h^{(2/3) + c} \qquad (9)$$

Let $k = k_m k_w$, then

$$u = (1/w)\ (dq/dh) = k/(c + 1)^{5/3}\ [(2/3) + c + 1]\ h^{2/3} \qquad (10)$$

$$v_2 = k[h_2^{2/3}/(c + 1)^{2/3}] \qquad (11)$$

$$u/v_2 = [(2/3) + c + 1]/(c + 1) \qquad (12)$$

If $c = 1/2$

$$u/v_2 = 13/9 = 1.444 \qquad (13)$$

For a common parabolic section, Seddon's wave-velocity is (13/9)v.

Proofs of his formula given by Seddon and others, and that given above, are neither rigorous nor wholly satisfactory because: (1) They neglect the effect of the initial flow and stage; (2) variation of slope is not taken into account although the slope often varies considerably in the vicinity of the crest, being less behind, and greater than the normal slope in front of the crest; (3) as the crest-profile is curved, dS is not identical with $[(dh_1 + dh_2)/2]\ wd\ell$ but is somewhat greater. Neither has the formula been tested by any adequately controlled experiments.

Forchheimer's formula--Using the Chezy formula and assuming a wide rectangular channel, Forchheimer [3] published in 1904 an equation for wave-crest velocity, which can easily be reduced to the form

$$u = (1/w)\ [(q_2 - q_1)/(h_2 - h_1)] \quad (14)$$

This equation fails when h_1 and h_2 are equal but, on the other hand, in the vicinity of the crest of a long wave, $(q_2 - q_1)$ may be taken as dq, and $(h_2 - h_1)$ as dh, and Forchheimer's equation reduces to identity with Seddon's. Apparently Forchheimer's formula can therefore be applied either to a river flood-wave or to an abrupt change of stage from h_1 to h_2.

For a wide rectangular channel, by Manning's formula, $q = w\ k_m h^{5/3}$ and $v = k_m h^{2/3}$. Hence

$$u = (1/w)\ [(q_2 - q_1)/(h_2 - h_1)] = k_m\ (h_2^{5/3} - h_1^{5/3})/(h_2 - h_1)$$

and

$$u/v_2 = (h_2^{5/3} - h_1^{5/3})/h_2^{2/3}\ (h_2 - h_1)$$

Let $\rho = h_1/h_2$, then

$$u/v = (1 - \rho^{5/3})/(1 - \rho) = 1 - (h_1/h_2)^{5/3}/(1 - h_1/h_2) \quad (15)$$

Generalized wave-velocity equation--The following equation is derived in the same way as that of Forchheimer excepting that it is based on the Manning formula, $v = (1.486/n)\ h^{2/3}\sqrt{S}$ and applies to any cross-section where the width is a simple power-function of the depth, $w = k_w h^c$, that is, to any type of parabolic cross-section ranging from a wide rectangle where c = 0, through parabolas with exponents c = 0 to c = 1.0, the latter corresponding to a triangular stream-section.

The section area is

$$A = \int_0^h w dh = [k_w/(c + 1)]\ h^{(c + 1)}$$

and the hydraulic radius is, nearly, (A/w), or R = h/(c + 1); also A = hw/(c + 1),

$$v = k_m R^{2/3} = k_m/(c + 1)^{2/3}\ h^{2/3} \quad (16)$$

where k_m is as given by (3).

From the equation of continuity, neglecting changes of channel-storage due to channel-friction, for any particular section with $w = k_w h^c$, the mean depth is h/(c + 1) and the change of channel-storage in any length $d\ell$ during the passage of the crest-stage is

$$A_1\ (u - v_1) = A_2\ (u - v_2)$$

$$A_1 u - A_1 v_1 = A_2 u - A_2 v_2$$

$$u = (A_2 v_2 - A_1 v_1)/(A_2 - A_1) = (q_2 - q_1)/(A_2 - A_1) \quad (17)$$

For a parabolic section, A = hw/(c + 1) and

$$u = (c + 1)\ (q_2 - q_1)/(h_2 w_2 - h_1 w_1) = (c + 1)\ (q_2 - q_1)/[k_w(w_2 h_2^{c + 1} - w_1 h_1^{c + 1})] \quad (18)$$

but

$$q = k_w k_m\ h^{(5/3) + c}/(c + 1)^{5/3} = Av \quad (19)$$

This gives

$$u = k_m/(c + 1)^{2/3} \times [(h_2^{(5/3) + c} - h_1^{(5/3) + c})/(w_2 h_2^{c + 1} - w_1 h_1^{c + 1})] \quad (20)$$

For a wide rectangular section c = 0 and $w_2 = w_1$, (19) gives $u = (1/w)\ [(q_2 - q_1)/(h_2 - h_1)]$ the same as by (14). This in turn is identical with Seddon's formula for this case if $(q_2 - q_1)$ = dq and $(h_2 - h_1)$ = dh.

Equation (20) may be used to compute u directly but it is convenient to express u as a ratio to v_2.

The stage-discharge relation, or rating curve, for many natural stream-channels can be expressed by an equation of the parabolic form, $q = k_q h^m$. From (17)

$$u = (q_2 - q_1)/(A_2 - A_1)$$

From (19)

$$q = k_w k_m h^m/(c + 1)^{5/3}$$

and since A = hw/(c + 1)

$$u = k_w k_m \,(h_2^m - h_1^m)/(c + 1)^{2/3}\,(h_2 w_2 - h_1 w_1)$$

But $w = k_w h^c$, therefore

$$u = [k_m/(c + 1)^{2/3}]\,[(h_2^m - h_1^m)/(h_2^{c + 1} - h_1^{c + 1})$$

Let $\rho = h_1/h_2$. Then, since [m - (c + 1)] = 2/3

$$u = [k_m/(c + 1)^{2/3}]\,[(1 - \rho^m)/(1 - \rho^{(c + 1)})]\,h^{2/3}$$

But from (18)

$$v = [k_m/(c + 1)^{2/3}]\,h^{2/3}$$

and

$$u/v_2 = (1 - \rho^m)/(1 - \rho^{(m - 2/3)}) = [1 - (h_1/h_2)^m]/[1 - (h_1/h_2)^{(m - 2/3)}] \qquad (21)$$

This gives the wave-crest velocity-ratio in terms of the depths and the exponent in the stage-discharge equation. This and equation (17) are the simplest general forms of equations for crest-velocity of channel-waves subject to friction-control.

The reason for the failure of the Seddon and Forchheimer equations to give correct results for natural channels or narrow experimental channels is apparently due to the following facts:

(1) The ratio (u/v_2) is a function of the exponent m in the rating-curve equation, $q = k_q h^m$.

(2) The ratio (u/v_2) varies with the ratio (h_1/h_2) for the same crest-stage h_2. For example, for a wide channel, m = 2. If h_2 = 16.0 feet and h_1 = 12, u/v = (1 - 9/16)/(1 - 3/4) = 7/4. If, as before, h_2 = 16.0 feet but h_1 = 4 feet, u/v = (1 - 1/16)/(1 - 1/4) = 5/4, or for the same crest-stage the crest-velocity is 40 per cent greater for a 4-foot rise on a 12-foot initial stage than for a 12-foot rise on a 4-foot initial stage.

The values of m in the discharge equation, $q = k_q h^m$, and c in the width equation, $w = k_w h^c$, for various parabolic sections, are given in Table 1.

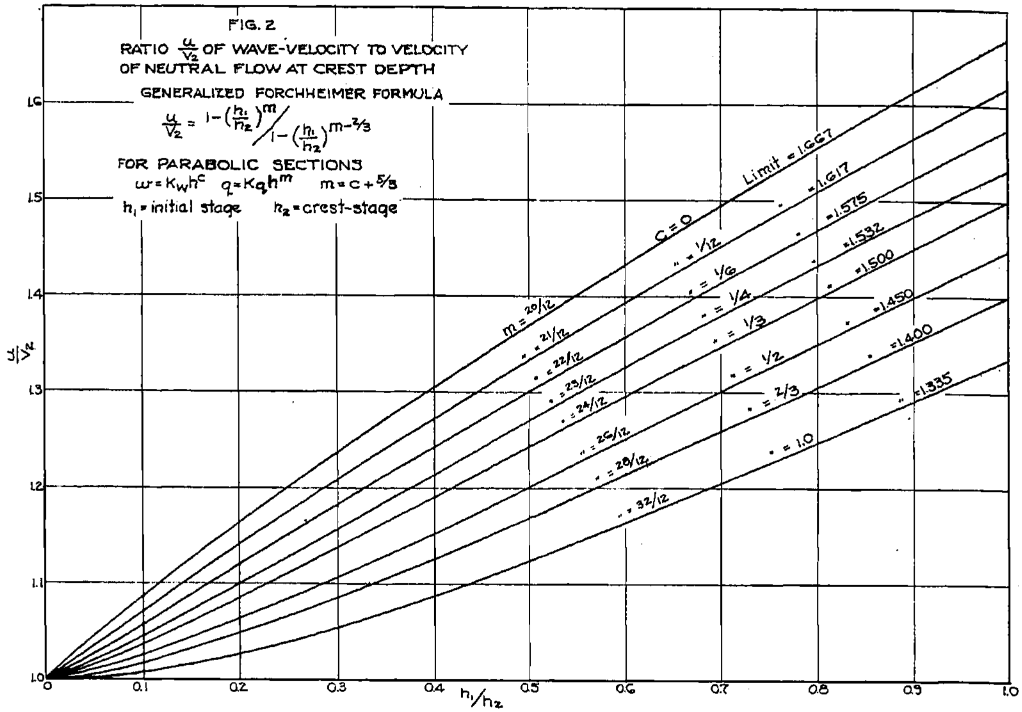
Table 1--Values of m and c

Section	c	m
Wide channel	0	5/3
Parabola	1/4	23/12
Parabola	1/3	2.0
Common parabola	1/2	13/6
Parabola	2/3	7/3
Triangle	1	8/3

Equation (21) gives (u/v_2) = 1.0 for h_1 = 0.

Figure 2 shows values of the ratio (u/v_2), or the ratio of the wave-crest velocity to the velocity of stable flow at crest-stage, for parabolic channel-sections of different types, in accordance with equation (21). The values of the ratio (u/v_2) start at unity for (h_1/h_2) = 0 for all types of channels and approach a limiting value as $h_1 \rightarrow h_2$. This limiting value is dependent on the form of channel cross-section and ranges from 4/3 for a triangular channel to 5/3 for a rectangular channel. As already noted, the ratio (u/v_2) = unity for h_1 = 0, or, for inflow to a dry channel, does not correctly represent the velocity of the front or toe of the wave.

Comparison of Seddon and generalized Forchheimer formulas--While equation (17), $u = (q_2 - q_1)/(A_2 - A_1)$ and Seddon's formula, u = (1/w) (dq/dh), become identical if $(q_2 - q_1)$ and $(A_2 - A_1)$ are differentials, the equations do not give identical results except for



infinitesimal waves. This suggests at once that the Seddon formula really applies only to small displacements of the water-surface. The argument has been made by Seddon and others that any long wave, such as a river flood-wave, may be considered as the summation of a series of successive infinitesimal increments, and that the Seddon formula therefore applies to all flood-waves.

For ordinary river flood-waves where there is an initial depth h_1 which is different from the crest-depth h_2, the velocity by Seddon's formula is that given by the ratio (dq/dh) at the crest-stage, and Seddon's formula does not take into account the effect of the initial flow. Undoubtedly the initial stage h_1 has some effect on the wave-crest velocity, a marked effect for instantaneous increments, and an effect decreasing as the wave-length increases, and possibly becoming negligible for waves of such great length relative to their height as ordinary flood-waves. It seems quite certain that given a wave of say 20 feet crest-stage, or h_2 = 20.0, the crest-velocity would be different for initial stages h_1 = 5 feet, 10 feet, and 15 feet, respectively, whatever might be the wave-length. Seddon's formula, however, gives identical crest-velocities for all these cases. On the other hand, equation (17) takes into account the initial stage h_1 and not only gives different crest-velocities from those given by Seddon's formula where the ratio (h_1/h_2) is different from zero, but for a given value of h_2, equation (17) gives a different crest-velocity for each different value of the initial stage h_1. Taking, for illustration, a hypothetical case of a river-channel 100 feet wide, with uniform slope, for which the stage-discharge relation is, in accordance with the Manning formula, $q = 100.0\ h^{5/3}$, the crest-velocities given by the generalized Forchheimer formula, for h_2 = 20.0 feet and h_1 having different values, are shown in Table 2.

Table 2--Velocities in feet per second by equation (17), for a channel 100 feet wide, with $v = 1.0h^{2/3}$ and $q = 100h^{5/3}$, for h_2 = 20.0 and for various values of h_1

h_1 =	1	2	4	6	8	10	12	14	16	18	20
u =	7.70	8.07	8.58	9.11	9.61	10.10	10.56	11.00	11.44	11.88	12.28

v = 7.37. u_s (Seddon) = 12.28 feet per second

It will be seen from Table 2 that the generalized Forchheimer formula gives crest-velocities which range from the velocity of stable flow for steady inflow to an initially dry channel, or for (h_1/h_2) = 0, to the velocity given by the Seddon formula for (h_1/h_2) = 1.0, or for a slight displacement. The change from one of these velocities to the other as (h_1/h_2) increases, is not precisely uniform but, to a fair degree of approximation,

$$U_F = v_2 + (u_S - v_2)\ (h_1/h_2)$$

where u_F and u_S are the wave-crest velocities by the generalized Forchheimer formula and by the Seddon formula, respectively.

It is of interest to note that for a steady release of water into an initially dry channel, both the Seddon and generalized Forchheimer equations give $(u/v_2) = 1.0$, which is the velocity of stable flow at the depth h_2; in other words, it is the velocity for complete friction-control. The author's experiments and those of Bazin and Darcy show that the velocity of the wave-front under these conditions is not v_2 but is evidently a function both of v_2 and $\sqrt{gd_2}$, indicating that the velocity of the wave-front is subject in part to momentum-control and in part to friction-control. At a sufficient distance back from the wave-front, where the flow has become steady, the velocity is subject to friction-control and is given correctly by the Seddon and Forchheimer equations.

In general, the stage-discharge relation at a given cross-section of a stream-channel can be represented by the equation $q = K_q h_2^m$. Hence $(dq/dh_2) = mKh_2^{m-1}$ but the velocity of normal or stable flow is given by $v_2 = Kh_2^{m-1}$. Consequently the ratio of the wave-velocity to the velocity of stable flow is equal to the exponent m. For different types of parabolic sections this ranges from 1.33 to 1.67.

For an infinitesimal wave or a small displacement wave, or, in the limiting case, where $(d_1/d_2) = 1.0$, Seddon's formula gives the same wave-velocity as for inflow to an initially dry channel if the final stable depth and velocity, d_2 and v_2, are the same. The generalized Forchheimer formula is in form indeterminate for this case, since $(q_1 - q_2)/(A_1 - A_2) \rightarrow 0/0$ as $(q_2 - q_1)/(A_2 - A_1)$ approaches zero. The graph (Fig. 2) shows, however, that in this case the wave-velocities by the generalized Forchheimer formula are the same as those given by Seddon's formula. This result does not appear to be correct in the light of the author's experiments and those of Bazin and Darcy, which show that the velocity of an infinitesimal wave or a slight displacement wave is accurately given by $u = v_2\ \sqrt{gd_2}$. Whether the Seddon and Forchheimer equations give correct wave-velocities for long waves under intermediate conditions remains to be determined.

Checking Seddon's and Forchheimer's equations--Table 3 shows the results obtained by the application of the Seddon and the generalized Forchheimer equations to some of the author's experiments [1]. In the case of these experiments, accurate stage-discharge relation curves were available for the experimental channel for each series of experiments, and the channel width was 5-5/8 inches, of rectangular cross-section. The wave-velocities by Seddon's formula were computed by determining the equation of the stage-discharge relation curve and finding therefrom the ratio (dq/dh). In general, the velocities by the Seddon and generalized Forchheimer equations are not materially different for the conditions of these experiments. Velocities by the Forchheimer equation are given for all of the experiments in the original paper [1]. The value of q_2 used in computing the velocities in Table 3 are those for stable flow after the wave has passed. It will be noted that with exception of one case, both equations give velocities much below the observed velocities. The observed velocities were, however, those of the toe of the wave in cases indicated by a star. The exception occurs in case of rectangular waves. For various reasons, pointed out in the original paper [1], the experiments on waves of initially rectangular profile are much less reliable than those on other forms of waves. Even in case of triangular waves used in these experiments, the ratio of wave-length to wave-height was relatively small compared with that occurring in case of natural flood-waves in river-channels. So far as the author is aware there are no laboratory-experiments on flood waves adequate to afford a reliable check on the validity of either the Seddon or Forchheimer equations, and it is evident that to check these equations adequately by laboratory-experiments requires an experimental channel of relatively great length. Pending the acquisition of experimental data for this purpose, it is desirable to undertake to determine the relation between actual flood-crest velocities in natural stream-channels and the crest-velocities as computed by the two formulas. This, however, involves practical difficulties owing to variations in channel-width, form of cross-section, variations in initial depth h_1, and owing also to the effect of lateral inflow. Furthermore, in order to apply either the Seddon or Forchheimer equation it is necessary to have available rating curves and average channel-widths representative of the stream-reaches within which the formula is to be tested. Adequate data of stream-widths are not usually available. It appears, however, that data have been obtained in certain cases adequate to permit a study to be made of the validity of the Seddon and Forchheimer equations. In particular:

(1) Flood-crest velocities in the Rio Grande River--See discussion in Water Bulletin 6, International Boundary Commission, United States and Mexico, State Department, Washington, D. C.,

Table 3--Wave-velocities by Seddon and generalized Forchheimer equations compared with observed wave-velocities of Horton's experiments

No. of Exp. (1)	Slope (2)	Seddon u(c) fps (3)	Forchheimer u(c) fps (4)	Observed u fps (5)	No. of Exp. (1)	Slope (2)	Seddon u(c) fps (3)	Forchheimer u(c) fps (4)	Observed u fps (5)
	Instantaneous increase			(a)		Gradual increase			(a)
4	level	1.88	2.23	4.45	9	level	1.77	2.12	4.19
4	level	3.69	2.31	4.01	7	level	3.87	2.28	3.82
2	.00102	1.96	1.82	3.23	4	.00790	4.24	3.12	4.79
3	.00102	2.14	2.09	4.44	20		3.00	2.38	4.18
3	.00186	2.41	2.32	4.50	Ratio to obs'd		0.716	0.569	
5	.00392	3.35	2.92	4.66		Gradual decrease			(b)
9	.00736	4.63	4.64	5.58	8	level	1.24	1.60	4.50
2	.00790	4.02	4.00	4.95	8	level	2.71	3.05	4.05
32		3.32	3.11	4.71	4	.00790	3.84	4.37	5.50
Ratio to obs'd		0.704	0.660		20		2.35	2.73	4.52
	Instantaneous decrease			(b)	Ratio to obs'd		0.520	0.604	
4	level	1.43	2.18	4.69			Rectangular		(b)
5	level	2.28	1.78	4.23	9	level	1.82	3.17	3.90
2	.00102	1.65	1.83	4.33	8	level	1.08	4.82	4.72
3	.00102	2.00	1.97	5.44	9	level	1.64	4.84	4.44
3	.00186	2.21	1.67	4.68	26		1.53	4.26	4.34
4	.00392	2.97	3.00	5.31	Ratio to obs'd		0.353	0.980	
10	.00736	3.93	4.36	5.73			Triangular		(a)
2	.00790	3.42	3.55	5.91	7	level	1.90	2.55	4.35
33		2.76	2.88	5.28	8	level	3.73	2.51	3.91
Ratio to obs'd		0.523	0.545		15		2.87	2.53	4.12
					Ratio to obs'd		0.697	0.614	

(a) Velocity of toe. (b) Velocity of crest. (c) Computed for stable depth.

1936; also Special flood report, flood of September and October 1932, Rio Grande River, International Boundary Commission, State Department, Washington, D. C., 1933.

(2) North Branch Platte River, flood of May-June, 1936--Data in manuscript form are available in the files of the United States Geological Survey, with excellent rating curves at numerous gaging stations, but additional data of stream-widths are needed.

(3) Delaware River--Data of flood-graphs, stream-widths, etc., are available in the author's files for a study of the applicability of the Seddon and Forchheimer formulas to flood-crest velocities in the Delaware River.

(4) In addition to the above it appears certain that there is a large amount of data on flood-crest velocities in the files of the United States Geological Survey and United States Weather Bureau which could be utilized for the purpose of testing the validity of the Seddon and Forchheimer equations but additional data of stream-widths would probably have to be obtained.

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