VIRTUAL CHANNEL-INFLOW GRAPHS

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Basis of virtual channel-inflow graph--What may be called a "virtual channel-inflow" graph" can in many cases readily be derived from certain properties of the channel-outflow graph (for the sake of brevity this will be designated a virtual-inflow graph or V-I graph).

If a given channel-inflow graph is transposed in time so that the channel-inflow begins simultaneously with the outflow at a given station, then the following four relationships will hold true between the transposed channel-inflow graph and the outflow-graph which it produces (see Fig. 1):

(1) Channel-inflow ends at the point of contraflexure P_2 on the recession side of the channel-outflow graph.

(2) The inflow- and outflow-graphs have a common point at the maximum outflow.

(3) Channel-inflow begins at the beginning of channel-outflow.

(4) The areas under the channel-inflow and outflow-graphs are equal and each represents the total surface-runoff.

These four rules make it possible to construct a triangular or trapezoidal channel-inflow graph which will give the same total runoff, the same maximum runoff at the same time, and will have rising and recession-characteristics nearly the same as those produced by the actual channel-inflow graph. This may be called the virtual channel-inflow graph. Its accuracy in any given case can readily be checked by using it as a basis for computing the channel-outflow graph in the manner later described.

The triangle abd (Fig. 1) meets the requirements listed. The point d is under the point of recession-contraflexure P_2 . The inflow recession-line starts at b and passes through the out-flow-crest c. The rising side of the V-I graph starts at beginning of runoff at a and is drawn to such a point b that the area abd is equal to the area A under the outflow-graph, excluding ground-water flow. Since the time base b = ad, the crest-height or altitude of the triangular V-I graph is given by the equation

 $I_{p} = 2A/b$

$$\mathbf{A} = \mathbf{b}\mathbf{I}_{\mathbf{g}}/2 \tag{1}$$

where I_{σ} is the maximum point on the V-I graph.

or

In addition to the four requirements listed, the V-I graph for a normal single-crested stream-rise has two other characteristics:

(a) The crest of the V-I graph occurs at about the same time as the point of contraflexure P_1 on the rising side of the outflow-graph. This requirement is approximately met by the triangle abd (Fig. 1).

(b) While the rising side of the outflow-graph is nearly always steeper than the recessionside, the reverse is usually true for a channel-inflow graph. For the latter the slope of the rising side is more gradual because of the fact that in the early stages of surface-runoff a large part of the rainfall-excess goes to build up surface-detention, and surface-runoff can only occur at the rate determined by the depth of surface-detention, which sustains the overlandflow. After rainfall-excess ends, the remaining channel-inflow is derived solely from surfacedetention and this is quickly exhausted by the combined effects of surface-runoff and infiltration. Hence the V-I graph drops off rapidly, the slope of the recession-side is steeper than that of the rising side, and the recession-interval on the channel-inflow graph is less than the interval of rise, as shown by the graph abd (Fig. 1).

Limiting forms of virtual-inflow graphs--The triangular V-I graph abd (Fig. 1) is not, howsver, the only form of channel-inflow graph which meets the four fundamental requirements. Neglecting the two secondary requirements, any figure having a base ad and a recession-side which falls on the line bcd and which has an area A equal to that of the outflow-graph, meets the four primary requirements for a V-I graph. Since the outflow-rate cannot exceed the inflow-rate, the altitude of the inflow-graph cannot be less than that of the outflow-graph. Also the rising side ab must either be vertical or slope upward to the right.

In the limiting (but improbable) case where the rising side of the inflow-graph is vertical, the four requirements are met by a trapezoidal inflow-graph, such as aefd. If a is the top width

of a trapezoidal V-I graph, then

$$A = I_t(a + b)/2$$
 (3)

where I_t is the crest-height for the trapezoidal graph. Since the maximum outflow-rate cannot exceed the maximum inflow-rate, the minimum possible value of I_t is the maximum outflow-rate, q_e .

If S is the slope of the recession-side of the V-I graph, in terms of time per unit-difference of vertical height or inflow-rate, then

$$S = (b - t_g)/I_g$$
(4)

where t_{σ} is the crest-time on the triangular V-I graph, measured from the point a.

In the limiting case of a vertical rise, the trapezoidal graph must have a height ${\rm I}_{\rm t}$ such that

$$[b + t_{g} + S(I_{g} - I_{t})]I_{t}/2 = A$$
(5)

or

$$[CI_{+} - (SI_{+})^{2}]/2 = A$$
(6)

where $C = (b + t_g + SI_g)/2$.

Hence

$$I_{t} = C/S + \sqrt{C^{2}/S^{2} - 2A/S}$$
(7)

from which the height of the trapezoidal graph of minimum crest-height can be determined. This is shown by the graph aefd of Figure 1.

Any V-I graph having a form between the limiting cases of the triangle abd and the trapezoid acfd meets the four primary requirements. Channel-inflow, however, never begins abruptly at full intensity and the rising side of the inflow-graph is never vertical. The most probable form of the V-I graph is that having a crest somewhere between the crest b of the triangular graph and the crest of the minimum trapezoidal graph. The mean of the limiting triangular and trapezoidal forms is not, however, the most probable or most usual form of inflow-graph. A study of many inflow-graphs shows that for a rise produced by a single continuous period of



FIG. 1 - VIRTUAL CHANNEL - INFLOW GRAPH

FIG.3 -EFFECT OF USE OF V.I. GRAPHS BOUNDED BY STRAKHT LINES

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rainfall-excess, the inflow-crest approaches the simple triangular form (abd, Fig. 1) so closely that in most such cases this form may be used without sensible error.

The method as thus far described applies to a normal outflow-graph such as is produced by a single period of rainfall-excess on the drainage-area.

In case of a double-crested or multiple stream-rise, the equivalent channel-inflow graph for each rise should be determined separately. The channel-outflow for each rise can easily be separated in most cases, since the recession-arc for the part of the first rise which continues after the second rise begins, can be obtained by transferring the recession-arc for the second rise to the first rise, since the quantity of channel-storage and the relation of channel-storage to outflow is the same for the same outflow-rates in both cases. The graph of the second rise can then be platted separately and the equivalent channel-inflow graph determined therefrom.

It will be noted that the V-I graph always begins at the beginning of the rise at the gaging-station. This is one reason why it is called a virtual inflow-graph. It is the channelinflow graph displaced in time so that it begins when the outflow-rise begins. It therefore eliminates the vexed question of time of channel-transit. The actual inflow-graph may of course begin and end earlier.

The outflow-graph represents the channel-inflow graph as modified by channel-storage but this modification cannot begin until the channel-storage becomes effective at the outlet or gaging-station. The V-I graph is in effect the same as if the channel-storage was all transferred to a reservoir just upstream from the gaging-station, the reservoir having the same capacity and the same stage-outflow relation for both rising and receding stages as the actual channel-storage.

The V-I graph represents correctly the sequence and magnitude of channel-inflow but not its actual time of occurrence. This statement is of course subject to the qualification that the V-I graph is made up of straight lines instead of having the more-or-less curved outlines of the channel-inflow graph.

Effect of form of outflow-graph on form of inflow-graph--While an outflow-graph of normal form can be accurately reproduced from a triangular or trapezoidal inflow-graph, it is obvious that sudden variations of rain-intensity may produce outflow-graphs that are not normal and inflow-graphs that are not triangular or trapezoidal. The form of the inflow-graph is reflected in that of the outflow-graph.

Referring to Figure 2, there were evidently abrupt changes of inflow-intensity at times t_1 and t_2 . If, now, a line aefg is drawn, composed of segments having slopes proportional to those of the corresponding segments of the outflow-graph, this line can be swung to the right or left until it is adjusted to such a position ae'f'g'h that the area under it equals the area abcdk. This will be a close approximation to the channel-inflow graph.

Effect of use of triangular instead of curvilinear graphs--The actual channel-inflow graph is made up of curvilinear surface-runoff graphs for individual sub-areas and has curved boundaries. That these boundaries depart little from the triangular or trapezoidal form is easily seen from Figure 3. The actual graph must start at a, pass through c' and end at d, and have the same area as the triangular or trapezoidal graph; also it must be such that the channelstorage is the same at the outflow-crest time c'. Hence the excesses and deficiencies of area under the true and linear graphs must balance between a and c', also at c' and again at d. This requirement permits of but small departures of the curvilinear graph from the triangular or trapezoidal graph and <u>vice versa</u>, and these departures are alternately positive and negative and have little effect on the computed outflow-graph.

Determination of channel-storage by the use of virtual-inflow graphs--Thus far proof of the Validity of the V-I graph rests solely on the fact that it meets the four primary requirements fixed by the hydraulic characteristics of the outflow-graph. More satisfying proof of the validity of the V-I graph is obtained by comparing actual outflow-graphs with those derived from the V-I graphs.

Channel-storage during recession, after surface-runoff or channel-inflow ends, can easily be determined by the method developed by the author [see l of "References" at end of paper]. Since there is no inflow, the channel-storage at any given point on the net recession-graph sub-sequent to the point P_2 (Fig.1) must necessarily be equal to the total subsequent channel-outflow. Hence the channel-storage can be obtained by summation backward of the area under the recession-curve of the outflow-graph. This method cannot be applied during rising stages when there is

inflow. During channel-inflow the volume of channel-storage is the difference between the massinflow and the mass-outflow.

For hydraulic reasons channel-storage for a given outflow-rate is usually greater during rising than during receding stages. Heretofore there has been no simple or satisfactory method for determining channel-storage during rising stages, since this determination requires the use of both the channel-inflow and the channel-outflow graphs. The V-I graph supplies the requisite inflow-graph, and the channel-storage and storage-outflow relation can be determined therefrom for rising stages by starting at the beginning of the rise and determining the difference between mass-inflow and mass-outflow down to the end of successive time-increments. Platting these quantities in terms of the observed outflow-rates gives a graph of the channel-storage-outflow relation for rising stages. Table 1 contains details of the computation of channel-storage for both rising and receding stages for the hydrograph shown on Figure 4.

To determine channel-storage for rising stages after the triangular V-I graph is derived, a series of time-intervals is set down from the beginning of the rise, as shown in column (2) of

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Table 1, and opposite each is placed the outflow-rate [column (3)] and the inflow-rate [column (4)] derived from the hydrograph. The difference [column (5)] is the rate of increase of channel-storage at the given time, and the mean of these rates at the beginning and end of the timeinterval [column (6)] multiplied by the time-increment (in this case 3,600 seconds) gives the increment of channel-storage for the given time-interval. By summation of these increments [column (8)], the total channel-storage for any given time from the beginning of the rise to the crest is obtained. For recession-stages the method of direct summation under the recession-graph is used, as described in a previous paper [1].

The dotted line on Figure 4 shows the march of channel-storage during the stream-rise and the insert on Figure 4 shows the channel-storage outflow-rate relation-curves for rising and receding stages. It will be noted that there is a hysteresis loop of channel-storage, the channelstorage being greater, at the same stage, for rising than for receding stages, and attaining a maximum at the outflow-crest. In the vicinity of the outflow-crest the channel-storage is sensibly the same for both rising and receding stages. Other examples of channel-storage curves are shown on subsequent figures.

<u>Construction of channel-outflow graph from virtual channel-inflow graph and channel-storage</u> <u>outflow-rate relations</u>--The following method of numerical solution of this problem applies to all forms of channel-inflow graphs and all types of channel-storage-outflow relations and to cases where the relation of channel-storage to outflow is not the same for rising as for receding stages. Subdividing the inflow-graph into equal time-intervals t and using subscripts 1 and 2 to designate conditions at the beginning and end of given time-increments, and letting I, q, and S represent inflow- and outflow-rates and channel-storage, respectively, then the storage-equation gives

$$(I_1 + I_2)t/2 = (q_1 + q_2)t/2 + (S_2 - S_1)$$
 (8)

Let $(I_1 + I_2)/2 = I_a$.

$$I_{a}t = (q_{1} + q_{2})t/2 + (S_{2} - S_{1})$$
(3)

S₁ is known and

$$S_2 = K_{cs} q_2^{1/M}$$
(10)

Let

$$2I_{a}t - q_{1}t + 2S_{1} = q_{2}t + 2K_{cS}q_{2}^{1/M} = \varphi(q_{2})$$
(11)

Compute the values of $\varphi(q_p)$ for

$$\varphi(q_2) = q_2 t + 2K_{cs} q_2^{1/M}$$
(12)

for a series of assumed values of q_2 . Computations of $\varphi(q_2)$ are carried out as shown on Table 2. The resulting $\varphi(q_2)$ -curves for rising and receding stages are given on Figure 5. To compute a point on the hydrograph, call the right-hand member of equation (12) Z and compute its value for the given time-interval. Then find on the $\varphi(q_2)$ -curve the value of q_2 which makes $Z = \varphi(q_2)$. This is the value of the channel-outflow at the end of the given time-interval and the beginning of the next succeeding time-interval. From the channel-storage curve take off the corresponding value of S₂. This becomes the value of S₁ for the next succeeding time-interval.

The computed channel-outflow graph is shown on Figure 4. The computations of this graph are carried out as shown on Table 3.

It is to be noted that $(S_2 - S_1)$ becomes negative after the outflow-crest is passed. When the computation has been carried through to the end of channel-inflow it can be continued to the end of channel-outflow, using the appropriate channel-storage-outflow relation. Longer timeintervals can usually be used in computing the recession-curve and new diagrams of channelstorage $\varphi(q_2)$ should be prepared. During the net recession-period following P_2 , the inflow is zero and the computation of Z is simplified. The values of $\varphi(q_2)$ remain the same as before.

<u>Validity of the virtual-inflow graph</u>--If the virtual-inflow graph is correctly drawn, the outflow-graph can be precisely duplicated therefrom in the manner described. If it is not correctly drawn, the outflow-graph computed therefrom will not agree with the observed outflowgraph. This is illustrated by Figure 6, which shows by a solid line the observed outflow-graph

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of the Fall Creek flood at Ithaca, New York, July, 1935. The inflow-graph correctly derived in the manner above described is shown by the triangle ABD, and the outflow-graph computed therefrom is in close agreement with the observed outflow-graph. The triangle ACE is an incorrect y-I graph with too long a time-base and too low a crest, and it will be noted that the outflowgraph computed therefrom does not conform closely to the observed outflow-graph.

It should be noted that the computation of an outflow-graph from a V-I graph is not a mere reversal of the process by which the inflow-graph is derived. It involves certain relations of channel-storage which are not a <u>priori</u> necessarily involved in the derivation of the V-I graph. At the crest-stage the channel-storage is at or near its maximum value and this represents the total accumulation of channel-storage during rising stages. It also represents the total depletion of channel-storage which must take place during recession, and these two quantities must be equal. If the V-I graph is correctly determined, these two quantities will balance and the computed outflow-graph will conform to the observed outflow-graph. If the V-I graph is not correctly determined, it will in general lead to a computed outflow-graph in which the channel-storage for rising and receding stages does not balance, and the computed graph will depart from the observed graph, particularly near the crest and during recession. There is therefore another requirement for a correct channel-inflow graph in addition to the four used in constructing the V-I graph, namely, it must balance the channel-storage for rising and receding stages.

Channel-storage, after surface-runoff ends, is determined by and only by the form of the outflow-graph and is independent of the assumed inflow-graph, whereas for rising stages, the channel storage-stage relation depends both on the assumed inflow-graph and the observed outflow-graph. Now, since (channel-storage + cutflow) = inflow, it follows that with an error in the inflow-graph there is an error in the channel storage-stage relation for rising stages, with the result that the storage-equation cannot belance for the recession-side of the outflow-graph, using the channel storage-stage outflow-relation derived therefrom. In other words, the areas under the inflow-and outflow-graphs can be equal if and only if the channel storage-outflow relations are correct, both for rising and receding stages, and they will be correct for rising stages

TABLE 2 - COMPUTATION OF \$9-CURVE TICGA RIVER NEAR ERWINS, N.Y.

RISE OF JULY 8, 1935

₹1 c.s.m	₹, t (t=3600 SEG.	E G.F. PERS.M	∲£1 =(2)-2(3)
(1)	(2)	(৪)	(4)

PISMA	SIDE
~	

1	3,600	180,000	363,600
z	7,200	340,000	687,200
4	14,400	559,000	1,114,400
6	21,600	709000	1.421600
8	28,800	825,000	1.678,800
10	36,000	930000	1,896,000
15	54,000	1,160,000	2 374,000
20	72,000	1390,000	2,852,000
Z5	90,000	1,610,000	3,310,000
30	108,000	1.850000	3808,000
35	126,000	2.080.000	4286,000
40	14,4000	2,310,000	4,764,000

RECESSION SIDE

		-	1
1	3,600	87,000	177,600
2	7,200	210,000	427,200
4	14,400	475,000	964,400
6	21,600	675,000	1,371,600
8	28,800	790,000	1.608,800
ю	36,000	870,000	1,776,000
15	54,000	1,100,000	2,254,000
20	72,000	1,320,000	2,7/2,000
25	90,000	1,560,000	3,210,000
30	108,000	1,820,000	3,748,000

only if the assumed inflow-graph is the true channel-inflow graph.

Uses of virtual channel-inflow graphs--The many practical applications of V-I graphs will not be discussed in detail but include the following:

(1) Determination of true time of transit--There has been much confusion regarding time of transit of surfacerunoff and channel-flow, or difference of time of occurrence of maximum rain-intensity and maximum channel-outflow. This interval is made up of three components: (a) Channelinflow lag, or difference of time of occurrence of maximum rain-intensity and maximum channel-inflow. This lag-interval is due to the reservoir-effect of surface-detention. (b) Time of channel-transit or difference of time of occurrence of maximum channel-inflow and maximum channel-storage at the outlet. This is chiefly the actual time of transit of water in the channel, either by hydraulic or wave-flow. but modified by the sequence of inflow. (c) Channel-storage reservoir-lag -- This is true reservoir-lag due to channel-storage and is the difference of time of occurrence of the crest of the V-I graph and the crest of the channeloutflow graph. Hitherto it has not been possible to segregate these components. The use of the V-I graph makes their separation possible.

(2) Law of overland-flow--The law of overland-flow can readily be determined for a simple sub-area tributary to one side of the reach of a stream if the surface-runoff is measured as in case of a runoff-plat experiment. The determination of the corresponding law of overland-flow for a larger drainage-basin made up of a large number of individual sub-areas, with different slopes and lengths of overland-flow, can be accomplished if the actual channelinflow graph is given, together with the areal rainfallexcess graph. It is possible to construct the areal rainfall-excess graph for a given drainage-basin from rainfalland infiltration-data if sufficiently complete, and since the rainfall-excess if sensibly equal to the surface-runoff, this gives the runoff-volume. The two graphs together make it possible to determine (1) the amount of surface-detention, (2) lam of overland-flow, (3) lag-interval due to surface-detention, or the interval between maximum rain-intensity and maximum channel-inflow.

(3) Flood-routing--Since the channel-outflow graph represents the channel-inflow graph as modified by channel-storage, a virtual-inflow graph provides a simple means of flood-routing in such a manner as to take into account changes in inflow-conditions. If, for example, virtualinflow graphs have been determined for existing conditions, together with the corresponding channel-storage graphs, then for a flood produced under modified conditions, the corrected channel-inflow graphs can be obtained for different points on the drainage-basin and these, in conjunction with the channel-storage relations, provide a ready means for determining the corresponding channel-outflow graphs.

(4) Construction of synthetic unit-graphs--The virtual channel-inflow graph provides a simple, direct means of constructing accurately the channel-outflow graph corresponding to a unit-storm of any duration and intensity. The resulting graph may be used as a unit-graph or distribution-graph for determining outflow-graphs in storms of other intensities. Unit-graphs can be derived in this way for any given amount of channel-storage and any given amount of initial inflow or ground-water flow.



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References

- [1] Robert E. Horton, Natural stream-channel storage, Trans. Amer. Geophys. Union, pp. 406-432, 1936.
- [2] Robert E. Horton, Natural stream-channel storage (second paper), Trans. Amer. Geophys. Union, pp. 440-456, 1937.
- [3] Robert E. Horton, Informal notes on the structure of the hydrograph and the relation between the channel outflow and channel inflow graphs; (processed); lecture given at Coshocton, Ohio, Experiment Station of the Soil Conservation Service, September, 1937. (The principles governing the construction of V-I graphs were first outlined in this paper.)



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5 -	6.		70.35	66.72	70.08	.5046	26.75	.0963	3.4000	3.8083	30.10	18500
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TABLE 3 - COMPUTATION OF CHANNEL-OUTFLOW GRAPH ~ TIOGA RIVER NEAR ERWINS, N.Y. RISE OF JULY 8, 1935

Voorheesville, New York

FLOOD-CREST REDUCTION BY CHANNEL-STORAGE

Robert E. Horton

<u>Conditions of similarity of channel-outflow graphs</u>-The hydrograph of a stream-rise or a channel-outflow graph represents the areal rainfall-excess graph for the drainage-basin as modified by (a) surface-detention, (b) channel-storage. It is desirable for many reasons to be able to derive a channel-outflow graph from the areal rainfall-excess graph, or at least to determine approximately the crest-outflow intensity therefrom. The virtual channel-inflow graph described in another paper [see 1 of "References" at end of paper] makes it possible to determine easily from an outflow-graph (a) the virtual channel-inflow graph, (b) the channel-storage characteristics, particularly at the time of the crest.

With reference to determining the crest-outflow rate from rainfall-data, two methods of procedure are available:

(1) The first depends on the fact that the storage-equation applies equally well to all forms of storage, whether channel-storage or surface-detention, or the two combined. It is therefore possible to use the same method as that used in determining virtual channel-inflow graphs but applying this directly to the areal rainfall-excess graph and thereby obtaining storage-characteristics for both channel-storage and surface-detention, which, once known, may be applied to the determination of crest-outflow intensities for other rainfall-excess patterns.