misuse of (i - q)-curves will be relatively large or insignificant depending upon the shape of the curve and the rainfall-pattern to which it is applied. In calculating runoff from either (i - q)- or f-curves the placement of the curve upon the rainfall-rate diagram may result in larger errors than those inherent in the approximate curves.

To calculate runoff accurately by any method is a complex, time-consuming job and the utilization of infiltrometer-data as an independent measurement of surface-runoff adds to the complexity of the calculations, but pays dividends in increased accuracy. Some time ago we analyzed a series of infiltrometer-experiments and our reluctance to using Dr. HORTON'S mass-line method as described in another publication was due to the amount of work required. To reduce the work, L. K. REINECKE modified this method and developed a procedure similar to that presented this morning. This experience with similar computations shows that Dr. HORTON'S simplified method is adaptable to "mass-production" determinations by relatively unskilled workers, and that it gives good results.

Dr. HORTON'S analysis of the phenomenon of infiltration is logical and is in accordance with field-observations obtained during a large number of actual tests. His method for determining infiltration-capacity curves involves two approximations. The first, substituting the f_c-rate in the equation $F_r = (f\ t_r/3)$, introduces an error in the subsequent calculation of δ_n, but as the differential, $d\delta_n$, is used to determine f this error nearly cancels and the resulting f-rates are substantially the same as those derived by using the calculated f, in place of f_c. This can be verified by treating the f-curve as a first approximation, reading from it the f for the given time, and inserting this rate in the member $(f\ t_r/3)$.

The second approximation, using δ_a and δ_n more or less interchangeably, ordinarily introduces no error because the depression-storage usually is about the same at the beginning of runoff as it is at the end of rainfall. This being true, depression-storage is the same on the rising and falling side of the hydrograph. There are frequent infiltrometer-tests, however, for which these depths are different. The large volume of depression-storage on irregular ground-surfaces such as exist under forest-cover may be so distributed that most of it occurs on the upper end of the plat. Under these conditions the storage-values calculated for the rising side of the hydrograph are inaccurate.

These two approximations, namely, the use of f_c and the substitution of δ_n for δ_a, cause a scatter of the derived points for the early part of the infiltration-capacity curve. The scatter reduces with increasing time so that a smooth curve, drawn through the points that do fall in line, can be extended backward through the scattered points to define an accurate infiltration-capacity curve.

We must realize that the storage-volumes on the plats have been derived in the past from measurements of rainfall and runoff only, and that an independent measurement of these volumes would extend our knowledge of infiltration-capacity. Research along this line is needed for a more thorough understanding of infiltrometer-data and data from small runoff-plats so that these may be applied more frequently in independent calculations of surface-runoff as a part of total runoff.

A SIMPLIFIED METHOD OF DETERMINING THE CONSTANTS IN THE INFILTRATION-CAPACITY EQUATION

Robert E. Horton

Elsewhere [see 1 of "References" at end of paper] the author has given a method for determining the infiltration-capacity curve or f-curve from an infiltrometer-experiment with constant rain-intensity, and has also shown [2] that such infiltration-capacity curves can be represented accurately by the equation

$$f = f_c + (f_0 - f_c)\ e^{-K_f t} \tag{1}$$

where f_0 = initial infiltration-capacity in inches per hour, f_c = final, constant infiltration-capacity in inches per hour, f = infiltration-capacity in inches per hour at time t in hours, and K_f is a factor which determines the rate of change of infiltration-capacity during rain. This equation expresses the march of infiltration-capacity during rain, for a given rain-intensity and drop-size, in terms of the three constants f_0, f_c, and K_f.

There is another important constant, t_c, which expresses the time in hours required for infiltration-capacity to drop from its initial value, f_0, to within one per cent of its constant or asymptotic value, f_c. The value of t_c is given by the equation

$$t_c = (1/K_f)\log_e[100\,(f_0 - f_c)/f_c] \tag{2}$$

The values of f_0 and f_c are derived directly from the infiltration-capacity curve.

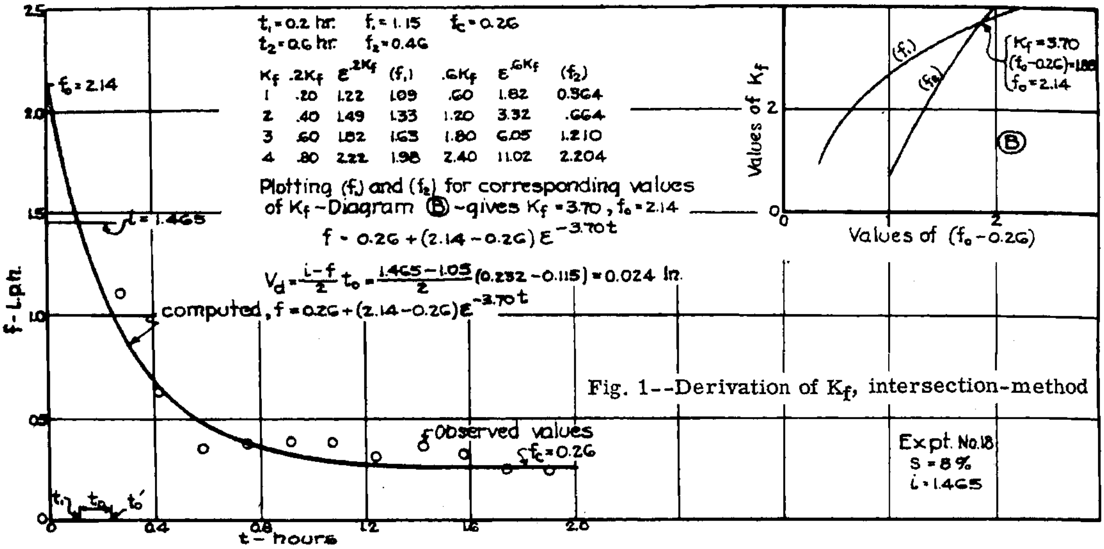

Fig. 1--Derivation of K_f, intersection-method

Heretofore two methods have been developed for determining K_f, namely, the method of intersections and the logarithmic method [3]. Figure 1 is an example of an f-curve and its equation derived by the intersection-method. Both these methods of determining K_f, while accurate, are somewhat involved and laborious.

A third method, that of taking the approximate value of t_c directly from the f-curve and then computing K_f from equation (3) derived from (2),

$$K_f = (1/t_c)\log_e[100\,(f_0 - f_c)/f_c] \tag{3}$$

has been tried but is not recommended. The value of t_c cannot be determined with sufficient accuracy by direct inspection from the f-curve. A large personal equation may be involved in its determination, and values of K_f derived in this way by two independent workers may easily differ so much as to give widely different f-curves.

The value of K_f so determined can be checked by computing the corresponding f-curve and comparing it with the original, but to secure agreement therewith may require the use of two or three trial values of f_c, and so the labor of determining K_f becomes even greater than where the method of intersections or the logarithmic method is used.

The following method of deriving K_f from an f-curve is much simpler than any of the three methods described, and gives results of a high order of accuracy.

The total or mass-infiltration, in inches depth, from the beginning of rain ($t = 0$) to the time t is

$$F = \int_0^t f\,dt \tag{4}$$

Substituting the value of f from (1) and integrating gives

$$F = f_c t + [(f_0 - f_c)/K_f]\,[1 - e^{-K_f t}] \tag{5}$$

The second term in the right-hand member of (5) represents the total mass-infiltration in time t taking place at rates in excess of f_c or, referring to Figure 2, it represents the area between the f-curve, abc, and the asymptotic line, od, for any time-interval t. Taking t equal to infinity, the

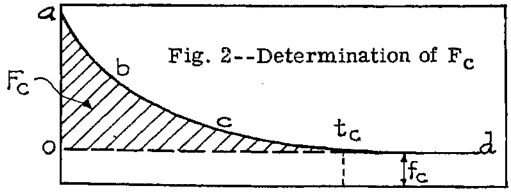

Fig. 2--Determination of F_c

right-hand term of (5) reduces to

$$F_c = [(f_0 - f_c)/K_f] \quad \text{and} \quad K_f = [(f_0 - f_c)/F_c] \tag{6}$$

This quantity F_c does not differ appreciably from the area oabc from $t = 0$ to $t = t_c$, and this area can be determined accurately from the f-curve by averaging the ordinates between zero and t_c or by planimeter-measurement. Hence the value of F_c is known and K_f can be determined at once by substituting this value of F_c in (6). Using the example shown on Figure 1, the value of F_c measured from the curve is 0.506 inch, $f_0 = 2.14$ iph, $f_c = 0.26$ iph, $(f_0 - f_c) = 1.88$ iph. From (6) this gives $K_f = 3.72$ as compared with 3.70 derived by the intersection-method.

In another example, with widely different conditions and from a different series of experiments, F_c derived from the curve = 0.2145, $f_0 = 2.045$, and $f_c = 0.145$. This gives $K_f = 8.88$, which is in good agreement with the value 8.75 derived by the intersection-method.

References

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[3] ROBERT E. HORTON, Derivation of infiltration-capacity curve from infiltrometer-experiments, U. S. Dept. Agric., Off. Land Use Coord., Flood Control, Washington, D. C., Mimeo., 18 pp., January, 1942.

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DISCUSSION

H. B. INGERSOLL (Soil Conservation Service, Washington, D. C.; communicated)--The paper presents a direct and useful method of determining the value of K_f. The other constants, f_0 and f_c, may be determined as described by the author in his analysis of NEAL'S experiment [author's reference 1].

These constants apply to the infiltration-capacity curve; not to the so-called (i - q)-curve, or to the (i - q')-curve, wherein the (i - q)-curve is corrected for surface-detention. To derive infiltration-capacity curves, estimates must be made for the values of surface-detention, depression-storage, and interception. Then, to calculate surface-runoff from an infiltration-capacity curve, it is necessary to use these values again as they are not reflected in the infiltration-capacity curve.

Where infiltration-capacity curves are being used in surface-runoff evaluation, values of K_f, f_0, and f_c are required to permit a rational means of averaging or compositing several observations. It is sometimes assumed that the composite infiltration-capacity curve, derived from several "runs" under the same conditions, is obtained more readily by compositing the infiltration-constants than by mere averaging of ordinates of various curves for individual "runs". But where surface-runoff is being estimated by "analogy", that is, by assuming that the hydrograph for an average "run" under given conditions is analogous in every respect to that for a typical unit of area, it is sometimes considered unnecessary to derive the infiltration-capacity curve. For this method surface-detention as observed is modified to appear at its relative time of generation, and the resulting curve, rather than the infiltration-capacity curve, is used. Such curves are usually different for different rainfall-intensities, but the infiltration-capacity curve is usually considered relatively constant, hence values of K_f, f_0, and f_c are real reflections of basic constants. On the other hand, these constants cannot be derived without estimating values of depression-storage, interception, and possibly other phenomena which in themselves are volumetric--and therefore depend on rainfall-intensity. For this reason, the ultimate effect of variations in rainfall-intensity must be considered, no matter which type of curve is used, before surface-runoff can be calculated from infiltrometer-experiments.