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STATISTICAL DISTRIBUTION OF DROP SIZES AND THE OCCURRENCE OF DOMINANT DROP SIZES IN RAIN [a]

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Abstract--In accordance with a model of a thunderstorm developed by the author [HORTON, publication pending], the ascending air current in a convective storm separates raindrops formed in the hail and snow stages into two components. The larger drops fall directly from the ascending air current, while the smaller drops are carried upward into the outflow layer and precipitated as peripheral rain.

In this paper various series of observations of raindrop size distribution are analyzed and it is found that the observed characteristics of statistical distribution of drop sizes in different parts of the storm and for rain, sleet, and hail, are in agreement with the results which follow from the operation of the thunderstorm model described.

DEFANT [1905] suggested that most raindrops result from the union of drops of a given initial size, so that the drop volumes are approximately integral multiples of this size. It is shown that there is a tendency toward the occurrence of dominant drop sizes which are roughly multiples of a given initial size. This apparently results from the lateral coalescence of drops of the same size as they fall at the same speed.

Drop-size distribution--BENTLEY'S [1904] data of drop-size distribution in 51 storms and for samples taken from the east, central, and western portions of each storm are given in Table 1. The percentage of the total number of drops of diameter larger than the upper limit of each class size is given in the last column of the table. These data, together with similar data from Lenard's observations (see Table 2) and LANDSBERG and NEUBERGER [1938] for sleet are shown graphically on Figure 1. These distribution curves are closely similar in form but Bentley's shows much higher percentages of numbers of large drops. It seems certain that his drop sizes, which were determined from flour pellets produced by the drop, are considerable in excess of the true diameters, especially for large drops, which formed greatly flattened pellets. Note that Figure 1 shows drop-size distribution, not distribution by volume.

Table 1--Relative number of drops of different size limits (after BENTLEY [1940])

Range	Diameter of maximum size	Portion of storm sampled				Cumulative		
		East	Central	West	Average	Upper limit of diameter	Total number larger	Per cent of total
in	mm					mm		
1/30	0.847	15	20	21	18.7	0.00	134.7	100.0
						0.847	116.0	86.6
1/30-1/18	1.42	51	46	46	44.0	1.42	72.0	53.8
1/16-1/8	3.18	42	42	40	41.7	3.18	30.3	22.6
1/7 -1/5	5.08	26	23	23	24.0	5.08	6.3	4.7
2/5	...	4	8	7	6.3	...	...	..

The large portion of total volume contained even in a relatively small number of large drops is shown by Table 3. It requires only four drops per minute 1/5 inch in diamter, falling on one sq inch to produce a rain intensity of one in/hr, whereas it requires about 65 drops per minute of the ordinary average diameter, two mm, to produce the same result.

If the number of drops of different sizes in a rain is known, the percentage of the total volume of the drops having diameters greater than a chosen size d_c can be determined by summation of the products of the volume per drop and the number of drops of different diameters.

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Table 2--Number of raindrops of various sizes in nine showers[a]

Diameter mm	Diameter in	Volume mm³	1	2	3	4	5	6	7	8	9	Mean
			Number of drops per m^2 per second[b]									
0.5	0.019	0.066	1000	1600	129	60	0	100	514	679	7	454.3
1.0	0.039	0.523	200	120	100	280	50	1300	423	524	233	358.9
1.5	0.059	1.77	140	60	73	160	50	500	359	347	113	200.2
2.0	0.079	4.19	140	200	100	20	150	200	138	295	46	143.2
2.5	0.098	8.19	0	0	29	20	0	0	156	205	7	46.3
3.0	0.118	14.2	0	0	57	0	200	0	138	81	0	52.9
3.5	0.138	22.5	0	0	0	0	0	0	0	28	32	6.7
4.0	0.157	33.5	0	0	0	0	50	0	0	20	39	12.1
4.5	0.177	47.8	0	0	0	0	0	200	101	0	0	33.4
5.0	0.196	65.5	0	0	0	0	0	0	0	0	25	2.8
All sizes, total			1480	1980	486	540	500	2300	1840	2190	500	1312.9
Rate of rainfall mm/min			0.09	0.06	0.11	0.05	0.32	0.72	0.57	0.34	0.26	0.28

[a]After P. Lenard, Meteorological Glossary, p. 335.
[b]Numbers 1, 2, and 3, ordinary rains; numbers 4, 5, and 6, convectional and thunderstorm types; numbers 7, 8, and 9, heaviest, medium, and terminal periods in a rain of cloudburst intensity.

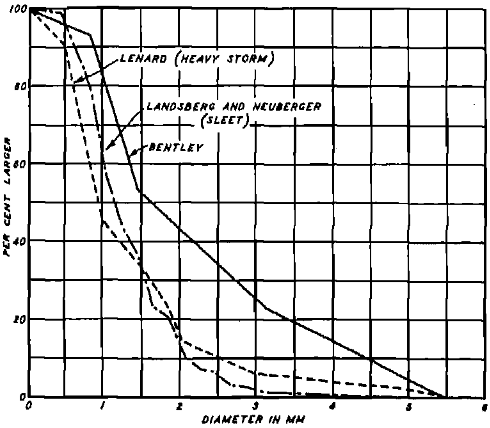

Fig. 1--Drop-size distribution in rain

Table 3--Diameter, suspension velocity, and volume of raindrops (after LAWS [1941])

Drop diameter mm	Drop diameter in	Suspension velocity m/sec	Suspension velocity ft/sec	Volume per drop mm³
0.5	0.019	2.30	7.54	0.066
1.0	0.039	4.15	13.61	0.523
1.5	0.059	5.50	18.04	1.77
2.0	0.079	6.58	21.58	4.19
2.5	0.098	7.40	24.27	8.19
3.0	0.118	8.02	26.31	14.2
3.5	0.138	8.50	27.88	22.5
4.0	0.157	8.85	29.03	33.5
4.5	0.177	9.08	29.78	47.8
5.0	0.196	9.25	30.34	65.5

From the data used in preparing Figure 1, curves showing the percentage of total volume of rain falling in drops exceeding different diameters were obtained, as shown on Figure 2.

If drop-size distribution were the same in the peripheral and central portions of the rain splash, then these exceedance percentages for a given storm would also represent the exceedance percentages for the entire volume of rain falling per unit of time over the rain splash area. As a rule there are variations in the drop-size distribution in different parts of the storm, larger drops predominating in the earlier, and smaller drops in the later, stages.

Bentley's data (see Table 1) comprise complete or nearly complete sets of samples from nine general rainstorms, 23 thunderstorms, and 19 rain showers. East and west portions of the storm, as given in Table 1 correspond approximately to the front and back of peripheral rain. Bentley concluded that "...in general the very small drops increase in number from the east (front) to the west (back) edge of the storm," and that drops of other sizes, each, as a rule, shows a progressive increase toward the center of a storm but then decreases toward the west portion. If only core rainfall occurred, then with only those drops falling from the core which have suspension velocities exceeding the ascent velocity in the part of the convection tube where they are formed, rain would consist of large drops during ascending gusts and small drops during lulls. Ascending gusts of different velocities would account for the occurrence of drops of all sizes in any part of the rain splash. Drops falling from the core may absorb other smaller and slower drops swept in

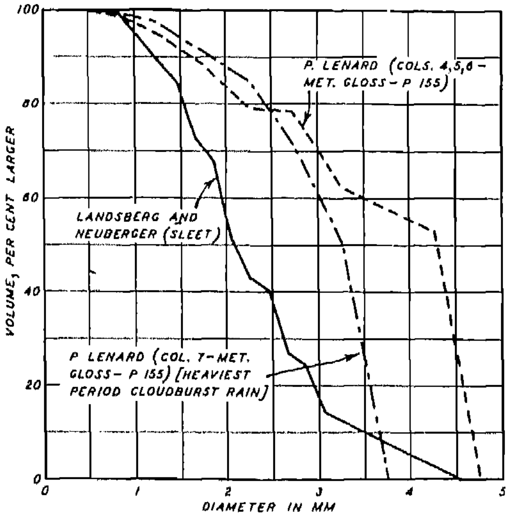

Fig. 2--Percentage of volume of rain drops exceeding a given diameter

their paths. This effect is probably small, for two reasons: (1) Core rainfall comprises initially larger drops. If they are over 5.5 mm in diameter they break into smaller drops. For drops between two mm and five mm in diameter, the maximum difference of suspension velocity is about two m/sec. With suspension velocity of eight m/sec, a larger drop of core rainfall will not overtake a smaller drop below it unless the larger drop is initially within a distance not more than one-fourth the height of fall at the start, and a much smaller distance if the drops are of appreciably the same size. (2) If the upper raindrop is much the larger, the smaller drop is likely to be pushed aside by the compression of air in front of the approaching larger drop.

Drops which later become peripheral rain are those arriving at the outlfow level with less than a certain critical size d_c, depending on the ascent velocity in the air current producing them. Drops of this critical size which rise to the top of the outflow layer cross the paths of all smaller coincident droplets. A drop which rises one-half way through the outflow layer before it is deflected outward crosses the paths of one-half of the coincident drops, and so on. Drops of the critical size can readily collide with other drops of the same or slightly smaller sizes in the outflow layer, as will be seen from Figure 3A, while drops of a given size cannot overtake drops of the same size in descending through the core.

Core rainfall apparently consists mostly of drops which have not been derived from the breaking up of larger drops nor from the coalescence of smaller drops. For the storm as a whole, all sizes of drops will occur as core rain because of the alternation of high and low velocities in wind gusts. At a given moment the size of drops comprising core rainfall will depend on the ascent velocity and the sequence of ascending wind gusts. It will be seen that drops occurring as core rainfall are mostly generated as such but must be of sizes that can fall through the ascending current of air.

Drops precipitated as peripheral rain are derived from smaller drops, either initial drops or drops resulting from the union of larger drops which have been carried aloft in the ascending air in the storm core. As they reach the outflow level they are generally smaller than drops of coincident core rain. Drops of peripheral rain near the inner boundary of the peripheral belt are often the result of cross-collisions in the peripheral ring. Larger drops of peripheral rain are concentrated near the inner edge of the peripheral belt and smaller and smaller drops occur at increasing distances from the core. These phenomena are illustrated by Figure 3A, which shows a cross-section of a convective thunderstorm.

The velocity of storm travel is governed chiefly by the velocity of the air in the cover layer. If this is greater than the component of wind velocity in the outflow layer in the same direction, then the peripheral belt of precipitation mnop, instead of being concentric with the convection tube, will be eccentric and will appear in cross-section somewhat as shown by the lines qrst. As a result, the belt of peripheral rain at the front of the storm will be narrower than at the back of the storm. The predominant drop sizes in a cross-section of the storm in the direction of travel will be somewhat as shown by Figure 3B. If Figure 3B is converted into a drop-size distribution curve, then the two summits will be combined and the drop-frequency curve will appear as shown by Figure 3C. This closely resembles the drop-size distribution of LANDSBERG and NEUBERGER'S [1938] drop-size data for sleet (see Fig. 4). The data were grouped by class increments of 0.2 mm diameter (see Fig. 4).

Since raindrops precipitated as core rain and peripheral rain orginate in different waves, a difference of drop-size distribution in the two cases is to be expected. Many more data on drop-size distribution are needed before a definite conclusion can be reached. The following examples indicate that there is a difference.

The frequency distribution equation

$$f = cd^2e^{-kd^2} \quad \ldots \quad (1)$$

gives (see Fig. 4) fairly good agreement with Landsberg and Neuberger's sleet drop-size data. This shows that frequency of drops of a given size is in this case the product of two factors, one an increasing, the other a decreasing, function of drop size. Observed drop frequency is of course that for rain as it falls, not necessarily nor probably the frequency of formation of drops in the convection core.

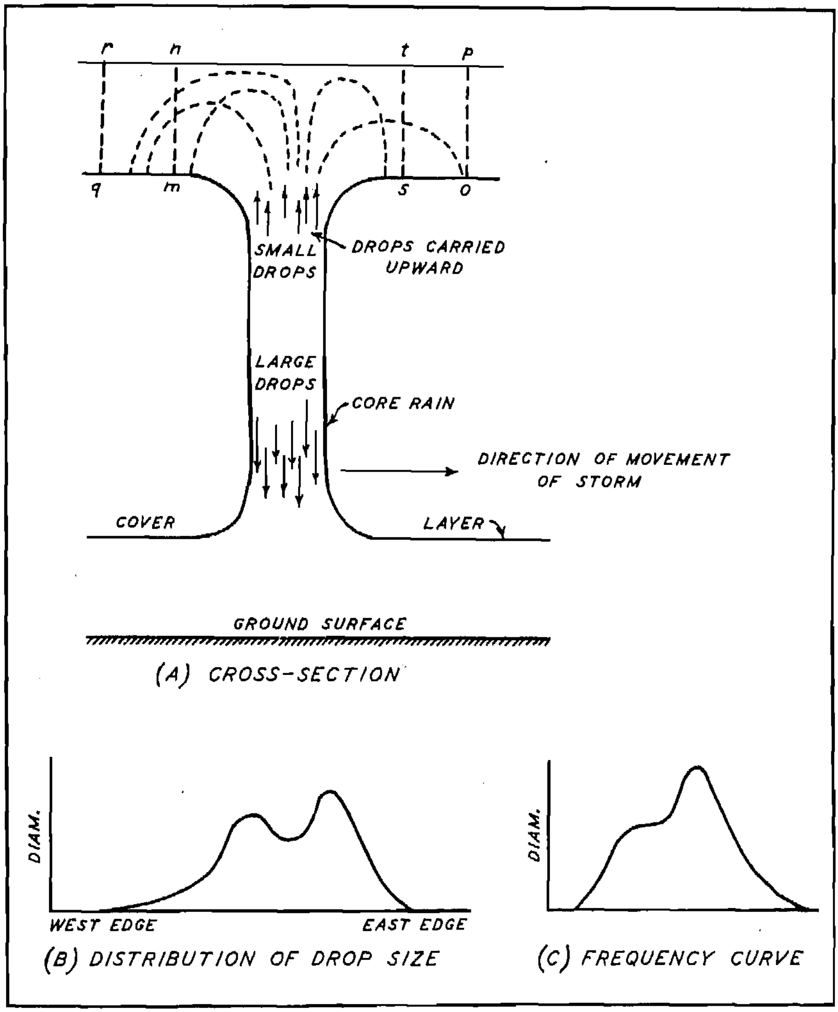

Fig. 3--Convective thunderstorm

Consider a layer of ascending air of vertical thickness dh within the core. Drops begin to form at a certain level. If there is no coalescence of drops, then all drops will occur more or less alike and all will be of about the same diameter after ascending through a given height. At a given moment the ascending air column in the core, taken as a whole, is made up of successive unit prisms at different heights and so contains drops of all sizes above the level at which drops begin to form.

If one of the two factors in (1) is the frequency of drop formation of a given size, then this factor must apparently be of the nature of an exhaustion process, since the size of the drops in a given unit prism dh will attain depends on the time during which the drop is exposed to condensation and those for drops produced in different ascending unit prisms will depend on the statistical distribution of gustiness in the ascending air. This follows from the fact that the accretion of condensation on a given drop is, other things equal, proportional to the product:

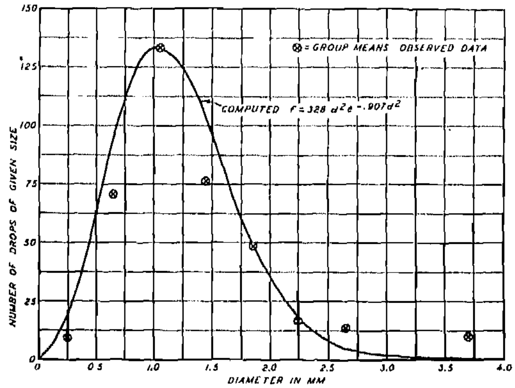

Fig. 4--Size distribution of sleet drops from LANDSBERG and NEUBERGER [1938]

(Average surface area of drop during ascent) × (average vapor pressure in the height of ascent) × (duration of ascent).

The last is governed jointly by the ascent velocity of the unit prism and the average terminal velocity of the drop during its time of ascent.

If the other factor is frequency of formation of a given drop size by union of two or more drops, this will increase as the cross-section area of the drop, that is, as the square of the diameter. As already shown, the requisite condition for union of drops of equal or nearly equal sizes is not well fulfilled by drops falling in parallel lines, as within the core. It is much better fulfilled for large drops falling through a medium in which the interspersed drops are moving horizontally or at a considerable angle with the path of the large drop. This condition is approximately provided in the peripheral outflow layer.

These considerations indicate that larger drops of peripheral rain from near the core and, to some extent, core rain, will conform to a two-factor frequency equation, such as (1). Peripheral rain occurring in small drops and falling from the outer part of the peripheral ring will conform to a single factor frequency equation

$$f = e^{-kd^m} \quad \text{(2)}$$

Both cases are represented by the general equation

$$f = cd^m e^{-kd^n} \quad \text{(3)}$$

Landsberger and Neuberger's data represent a case of drop-size distribution for rain (in this case sleet) in a shower with high ascent velocity and precipitation, chiefly from the peripheral belt. Drop-size distribution for core rainfall should also contain an exponential term, not necessarily the same power of diameter d but the other factor or d^2 term should be small or absent in the case of core rainfall. Drop-size distribution data in a proven sample of core precipitation are wanting.

The author believes that hail is formed chiefly by a process analogous to the formation of core rainfall but in vortex rings outside the core and without spreading in the outflow layer. If this is so, then the drop-size distribution for hail should be different from that for peripheral rain and similar to that for core rainfall. Data of frequency of hailstone sizes in India, collected by J. Elliot and analyzed by BROOKS [1944] can be represented by the following equation, derived by the author from another equation given by Brooks

$$F = 1.22\ e^{-1.22d} \quad \text{(4)}$$

F is the frequency of drops equalling or exceeding diameter d in inches. Since the relative frequency F cannot exceed unity, for d = 0.164 inch, (4) gives F = 1.0. The small hailstones considered are of this size.

If F' is the frequency of hailstones having diameters <d, then

$$F' = 1 - F = 1.22[(1/1.22) - e^{-1.22d^2}] \quad \text{(5)}$$

The relative frequency f of a specific size d is given by the differential of this equation, or

$$f = df' = [(1.22)^2\ e^{-1.22d}] \quad \text{(6)}$$

This is a single factor frequency equation representing a simple exhaustion phenomenon.

HOUGHTON and RADFORD [1938] determined the frequency distribution by volume for different drop sizes in clouds and fogs The resulting frequency curves are slightly skewed but closely resemble the curve of the normal or Gaussian law of error; in other words, these curves are not greatly different from those of the single term inverse exponential function

$$f = e^{-kd^2}$$

where f is frequency of drop volume, expressed as the product of number of drops and the cube of their diameter. Since raindrops must originate as minute drops or crystals of cloud or fog, this is at least indicative of drop-size distribution in the rain core before breaking up or recombination, and, as far as it goes, confirms the idea that there is a difference of drop-size distribution of core rain and peripheral rain and, in the latter case, a two-factor distribution equation is required to take into account the effect of the merging of drops by collisions.

Dominant drop sizes--DEFANT [1905] called attention to the occurrence of dominant drop sizes in rain and suggested that large drops are the result of successive collisions of drops of an initial size, the drop volumes forming a geometric series with terms 1, 2, 4, 8, ... A tendency toward predominance of particular drop sizes exists in the data of Bentley, Lenard, and, in particular, in the sleet drop sizes observed by Landsberg and Neuberger.

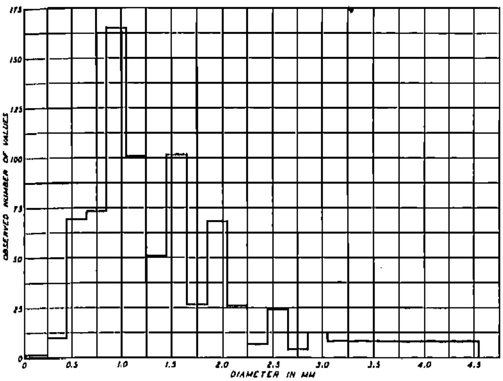

Fig. 5--Size distribution of sleet drops from LANDSBERG and NEUBERGER [1938]

Referring to Figure 5, there is a strong predominance of drops of about 1, 1.5, 2, 2.5, and 3 mm diameter. The corresponding relative volumes per drop are shown in Table 4. In this series a drop of a given diameter has, as a rough approximation, the same volume of two drops 1/2 mm smaller in diameter.

A drop of the critical size in the peripheral belt can overtake one or several drops of approximately the same size. Also it may combine with various smaller drops. Drop diameters increase as the cube roots of the drop volume. For example, increase of diameter by union of a drop of initial diameter d_c with a smaller drop of diameter d′ will be

d'/d_c	1.0	0.9	0.8	0.5	0.25	0.10
Relative diameter	1.26	1.24	1.22	1.14	1.08	1.03

The resulting diameters after union of several drops of the same size are

Number	1	2	3	4	5
Diameter	1.0	1.26	1.44	1.59	1.71
Increment of diameter	0.26	0.18	0.15	0.08	

Table 4--Volumes and frequencies of drop sizes

Diameter	Volume per drop (relative)	Frequency	Total volume (relative)
mm			
1.0	1.0	165	165
1.5	3.37	103	347
2.0	8.0	68	544
2.5	15.6	24	374
3.0	27.0	13	351

The final volume of a drop of the critical or nearly critical diameter after falling through the outflow layer will not be precisely an integral multiple of that of the drop corresponding to the number of larger drops combined. A factor controlling this result is the fact that if a drop of critical diameter d_c falls across a stream of smaller drops, the chance of collision with an independent drop is proportional to the square of its diameter.

From Figure 5 the maximum frequency is for dops one mm in diameter; the maximum volume is included in drops two mm in diameter. NEUBERGER [1942] has reviewed the earlier work of Lenard, Mache, Kohler, Defant, Niederdorfer, Hageman, and others on drop-size distribution and the occurrence of predominant drop sizes and gives a bibliography of the subject.

The Defant phenomenon is strongly marked in some drop-size series and little in evidence in other series. This would be true if it occurred chiefly in one of two types of rain, not in the other. Apparently it can occur in both core and peripheral rain but its occurrence is much more frequent and uniform in peripheral rain.

LANDSBERG and NEUBERGER [1938] aptly summarize the result of investigations of the Defant phenomenon, "If liberally interpreted, this can be taken as representing the proportions: 1:2:4:8:16. Other values are, nevertheless, frequently enough represented to show that these proportions may be a predominant feature of drop-size distribution but are by no means a lawfully required order."

It is well established that raindrop sizes and drop-size distribution are highly important factors in relation to soil erosion and they form a powerful tool for the quantitative determination of thunderstorm dimensions and characteristics in rain-intensity distribution. While many more data are needed for final conclusions, data at present available, viewed from various angles, show that observed facts in relation to drop sizes, their statistical and areal distribution in the rain splash, and the occurrence of dominant sizes, are consistent with results which follow from the convection tube model of a thunderstorm.

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