AN EXPERIMENT ON FLOW THROUGH A CAPILLARY TUBE

Robert E. Horton

A glass tube of 0.0736-inch or approximately two-mm bore was placed in a nearly horizontal position with one end slightly bent and immersed in a glass jar filled with water, as shown on Figure 1. For a given inclination of the tube the water flowed up the tube to such a distance from the outlet that the height of the capillary meniscus in the tube above the water-level in the jar was equal to the vertical height of capillary rise, in accordance with JURIN'S law. This distance L_c is given by the equation

$$L_c \sin a = c$$

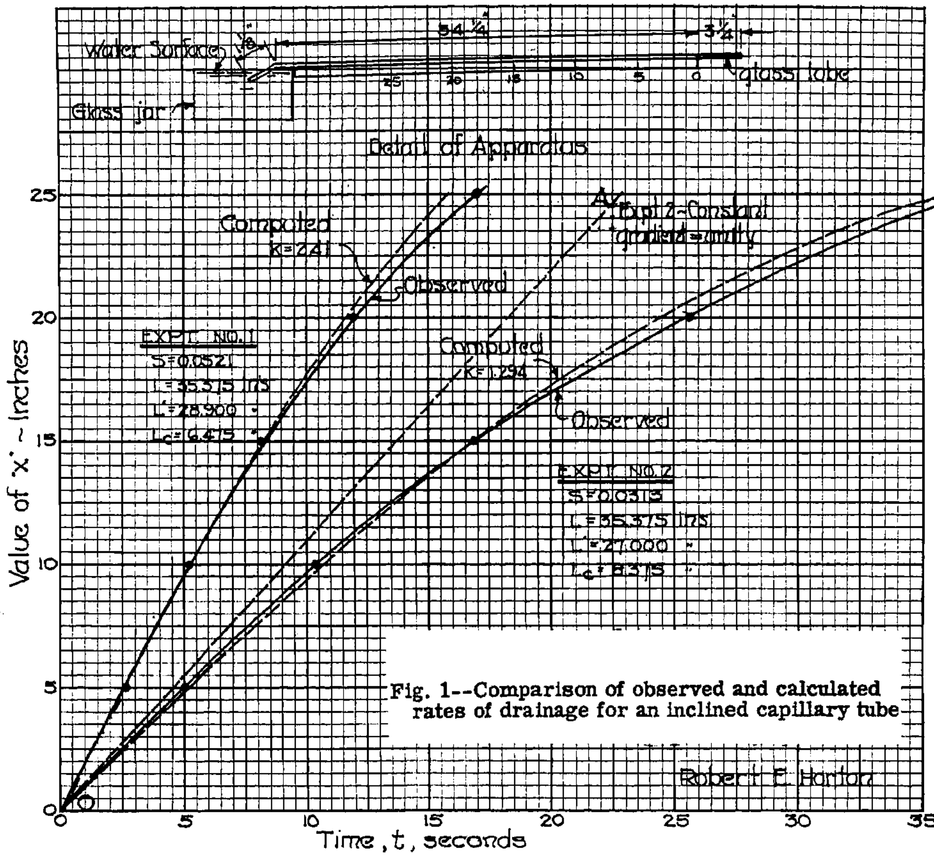

Fig. 1--Comparison of observed and calculated rates of drainage for an inclined capillary tube

where c = height of capillary rise in a vertical tube and a = angle of tube to the horizontal.

The tube was filled by suction, the upper end closed, and the tube then placed at a given angle and the inlet released. The water then flowed slowly down the tube, the rates of flow being determined by timing the passage of the meniscus along a scale adjacent to the tube.

In accordance with POISEUILLE'S law, as commonly applied in ground-water hydraulics, the rate of flow is proportional to the ratio of head to length of water-column in the tube. If the head is taken as the vertical distance from the capillary meniscus to the water-surface at the outlet, then, except for about one-inch immersion of the lower end of the tube, this ratio was constant for all positions of the meniscus in the tube in a given experiment, and the velocity of retreat of the meniscus should be constant.

The results of two experiments are shown on Figure 1, where t = time in seconds required for the capillary meniscus to retreat a distance x, in inches, from the zero-mark on the scale alongside the tube. It was found that the rate of retreat of the capillary meniscus decreased with its distance from the outlet, the flow ceasing when the meniscus had retreated to a distance L_c from the outlet, where it was at a height c above the reservoir. These phenomena are readily accounted for if it is assumed that the effective head inducing flow in this case is not the total head from the reservoir to the meniscus but the vertical head from a height c above the reservoir to the meniscus. On this basis an equation for the relation between the time t and the distance x travelled by the meniscus can readily be obtained as follows.

If x is the distance of the meniscus from zero of the scale at time t, and K the velocity at gradient unity (a quantity corresponding to the transmission-capacity of a soil), L the length of tube from zero of scale to outlet, and $L' = (L - L_c)$, where L_c is the length of capillary advance in the tube, then from POISEUILLE'S law

$$v = (dx/dt) = K_t\,[(L' - x)/(L - x)] \qquad (1)$$

$$K_t\,dt = [(L - x)/L' - x)]dx = [L/(L' - x)]dx - [x/(L' - x)]dx$$

On integration

$$K_t = x + L_c \log_e [L'/(L' - x)] \qquad (2)$$

Dotted lines on Figure 1 show the time-curves computed by equation (2) in two experiments. The slight excess of the observed over the computed times is at least partly due to the projection of the lower end of the tube into the reservoir. The experimental confirmation of equation (2) is very close. POISEUILLE'S law is confirmed if correctly applied. If, however, the capillary gradient was taken as unity, without deduction for capillary rise, POISEUILLE'S law gives a straight-line relation OA of time to distance x.

The experiments show that net or effective capillary head--not the gross capillary head--should be used in applying POISEUILLE'S law if correct results are to be obtained under the conditions of the experiment. The same thing is true in applying other flow-formulas, such, for example, as the MANNING formula for turbulent flow. The fact that the effective capillary head instead of the total capillary head should be used seems first to have been pointed out by HAGEN [see 1 of "References" at end of paper] and later by BOUSSINESQ [2] but without experimental confirmation.

PORCHET [3] performed many experiments on flow through soil-columns, using (1) soil-column fully submerged, (2) capillary surface within the soil-column, and obtained results similar to those obtained by the author in the latter case.

The results of one of PORCHET'S experiments are shown on Figure 2. Apparently PORCHET did not measure either the hydraulic head or the effective capillary head. He did measure the rate of outflow and the time, and found that the rate of outflow dropped off abruptly at the point α, Figure 2, at which the water-surface entered the soil-surface.

In subsequent discussion the following notation is used:

h_c = gross capillary head or difference between free-water level at the outlet and water-level in capillary menisci within the soil or capillary.

c = height of capillary rise.

h_n = net capillary head = $(h_c - c)$.

h_m = hydraulic or manometric head from free-water surface to free-water surface, as when measured in open wells.

b = minimum head required to initiate flow through extremely fine fissures, soil-interstices, or capillaries.

ℓ = length of substance through which flow takes place.

The term "capillary surface" is here used to describe the envelope of that portion of the capillary fringe in which the soil is completely or approximately saturated. In applying the equations homogeneous units must be used.

The author's experiments and those of PORCHET appear to be a particular application of a more general law. Numerous series of experiments have shown that flow through a given length ℓ of fine-textured substance, such as clays or rocks containing a large percentage of extremely fine

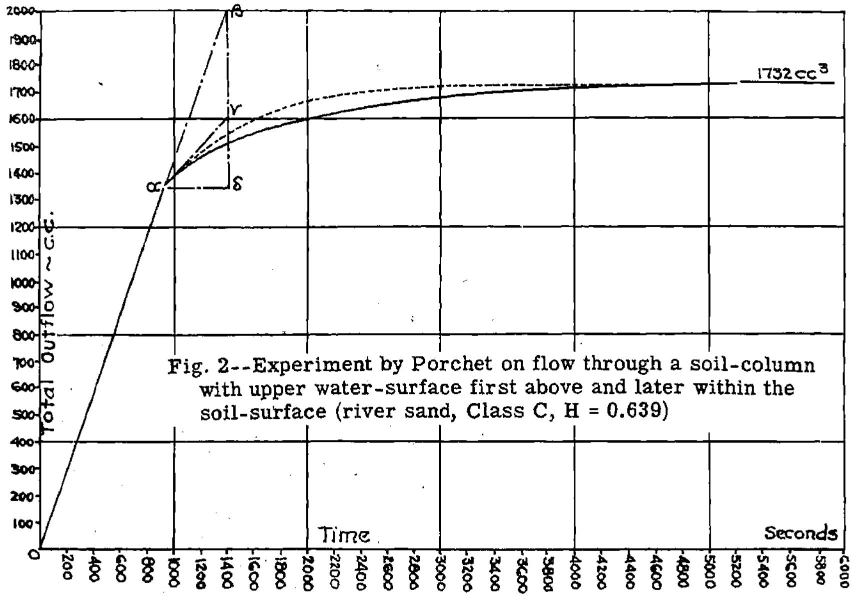

Fig. 2--Experiment by Porchet on flow through a soil-column with upper water-surface first above and later within the soil-surface (river sand, Class C, H = 0.639)

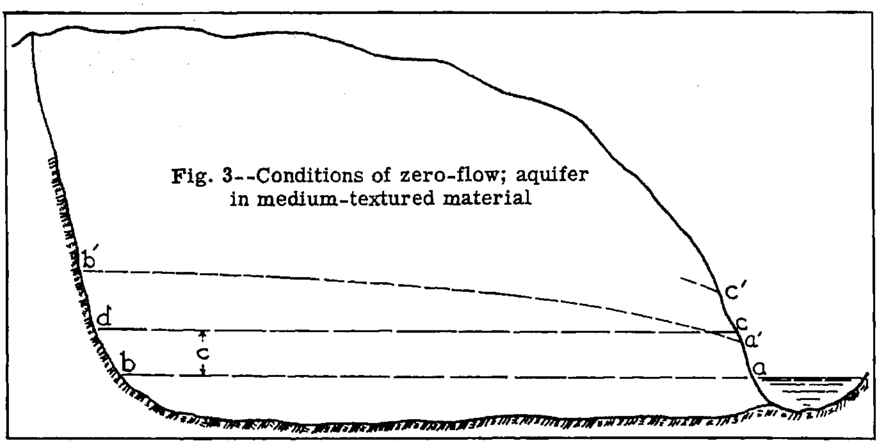

Fig. 3--Conditions of zero-flow; aquifer in medium-textured material

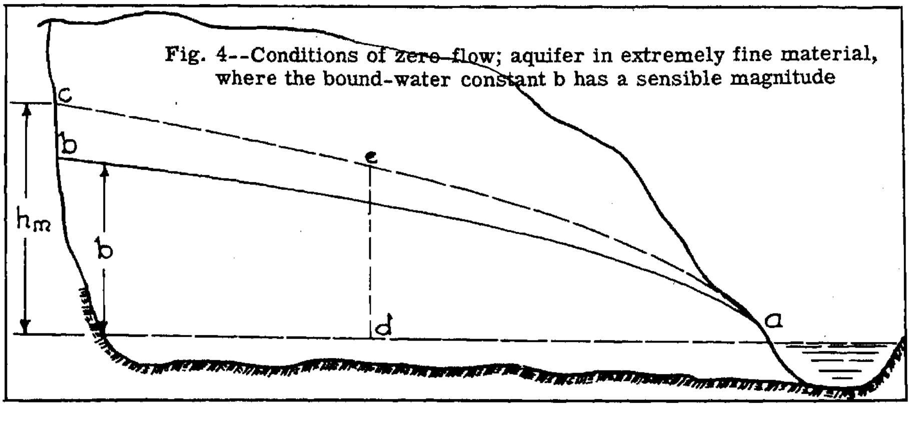

Fig. 4--Conditions of zero-flow; aquifer in extremely fine material, where the bound-water constant b has a sensible magnitude

pores or fissures, is zero unless the hydraulic head exceeds a minimum limit b [4]. This limit may be, and for dense substances often is, much greater than the length of flow through the substance. POISEUILLE'S law holds true for flow through such substances if the net effective hydraulic head (h_m - b) is used. Then

$$q = K_t [(h_m - b)/\ell] \tag{3}$$

where ℓ is the length of substance through which flow takes place.

Similarly, for flow in a capillary tube, under the conditions of the author's experiments, where b is negligible,

$$q = K_t [(h_c - c)/\ell] = K_t (h_m/\ell) \tag{4}$$

The identity of form of these two equations is suggestive. There is, however, a distinction between the two cases. The constants c and b are not identical. The constant b apparently corresponds to the effect of the "bound" water in the finer pores of the medium. For not too fine materials c may have values up to perhaps a few feet in natural soils, while b is sensibly zero. Flow then occurs for the slightest net capillary or hydraulic gradients, since in this case these two quantities are the same. In finer-textured materials the constant b is not zero.

Figure 3 shows conditions of zero-flow in an ordinary aquifer in not too fine-textured material where the constant b is zero or negligible. The line cd shows the capillary surface and the line ab the hydraulic grade-line, which in this case is horizontal. Water in the capillary stratum abcd below the capillary surface is in labile equilibrium with reference to horizontal flow. The section ac of the capillary fringe below the saturation-line is moist and glistens in sunlight but no outflow occurs. This is not a true seep-face. If the manometric surface is raised to a'b' a true seep-face aa' occurs below the intersection of the ground-water profile with the wall of the outlet, and water flows out of this seep-face. There is also a capillary face a'c' between the ground-water profile and the new capillary surface. Water in the layer abcd which did not flow under the first conditions assumed now partakes in flow toward the outlet.

Figure 4 shows conditions of zero-flow in a fine-textured aquifer where the constant b is of appreciable magnitude--in other words, where the capillaries are so small that all or most of the water is held as "bound" water by molecular forces until the hydraulic head exceeds the minimum value b for the given length of flow. In this case there would be no outflow with a ground-water profile ab but if the ground-water profile is raised to ac, flow would take place though not at a rate proportional to the manometric head h_m but at a rate proportional to (h_m - b). With the ground-water profile ac the water in the entire cross-section de would participate in the flow. Consequently for these conditions the flow would become zero or sensibly zero when the manometric head is less than b but when the manometric head is greater than b the rate of flow would increase much more rapidly than the manometric head increases. Thus it may happen that an aquifer which yields abundant water under ordinary hydraulic gradients may yield little or none if the hydraulic gradient falls to the minimum level (b/ℓ). Aquifers of this type are not likely to be of much practical importance if the texture of the matrix of the aquifer is uniform throughout. There are probably many instances of aquifers supplied by numerous rock-fissures where the size and frequency of the fissures decrease rapidly proceeding downward. Hudson River shale affords examples of rock of this type. Such aquifers may yield fairly abundant water when the ground-water profile is within the larger fissures close to the surface and the slope is great, but they may yield no water whatever when the profile has receded into the finer fissures and the slope has decreased to a value less than (b/ℓ), although in the latter case there may still be a definite positive hydraulic gradient or manometric head [5].

In such an aquifer the relation of head to flow-rate may follow POISEUILLE'S law closely when the head and yield are large but the flow may drop off much more rapidly if computed in terms of hydraulic head, with flatter slopes and lower water-levels.

Determinations in the laboratory of transmission-capacity are ordinarily made with a submerged soil-column, while in a natural surface-aquifer there is, as in the author's experiment, a capillary surface within the soil. The question arises whether transmission-capacities determined from a submerged soil-column are applicable to natural conditions. The author believes they are, for the simple reason that where there is a capillary surface within not too fine-textured soil, the hydraulic or manometric head $h_m = (h_c - c)$. Hence the manometric and effective capillary heads are in this case identical. The decrease in flow-rate in PORCHET'S experiments when the water-level fell within the soil-column was apparently caused by reduction of the effective head, not by a change of transmission-capacity of the soil.

References

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[3] M. PORCHET, Study on the underground flow of water, Ann., Documents Officiels, Jurisprudence - Rapports et Notes Technique (France et Étranger), Minis. l'Agric., Dir. Gen. des Eaux et Forêts, Fasc. 53, Paris, Imp. Nat., 1923, pp. 283-358.
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[5] MILLER-BROWNLIE, Sub-soil water in relation to tube wells, Indian and Eastern Eng., pp. 191-193, 1919; reprinted in Eng. and Contract., pp. 355-356, March 31, 1920.

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DISCUSSION

C. E. JACOB (U. S. Geological Survey, Jamaica, New York)--Mr. HORTON'S paper presents the results of an interesting experiment in non-steady uniform flow in a slightly inclined capillary tube, the lower end of which was submerged in a vessel containing water. The water was drawn up in the tube by applying suction at the upper end and then was allowed to flow back into the vessel under its own weight. It was found that the rate of flow decreased continuously with time, the meniscus finally coming to rest at a height c above the free surface, equal to the capillary rise appropriate to the particular capillary radius. "These phenomena are readily accounted for," states Mr. HORTON, "if it is assumed that the effective head inducing flow in this case is not the total head from the reservoir to the meniscus but the vertical head from a height c above the reservoir to the meniscus." Then after deriving the equation of flow and comparing the results of his experiments with the theory he concludes that "The experiments show that net or effective capillary head--not the gross capillary head--should be used in applying POISEUILLE'S law if correct results are to be obtained under the conditions of the experiment."

It would seem that Mr. HORTON'S definition of the term "head" is such that he finds it necessary to introduce additional terms in describing the behavior of the retreating meniscus. In the writer's opinion the terms "gross capillary head", "net capillary head", "effective capillary head", and "capillary gradient" are superfluous. The quantity which Mr. HORTON designates by h_m and which he calls the "hydraulic or manometric head from free-water surface to free-water surface, as when measured in open wells," is generally called simply "head" in ground-water hydraulics and is designated by h. This is the variable that is to be substituted in the POISEUILLE equation $v = -(Kh/\ell)$. It is defined as the potential energy per unit-weight of fluid at a given point and is expressed by $h = [z + (p - p_a)/w]$, where z is the elevation of the point in question with reference to an arbitrary geodetic surface, p is the fluid pressure, p_a is the atmospheric pressure, and w is the specific weight of the fluid. It is obvious that at the free-water surface in open wells $p = p_a$ and $h = z$, the elevation of the water-surface. Differences in head are therefore measurable vertically "from free-water surface to free-water surface . . . in open wells" as Mr. HORTON suggests. However, if the well is of extremely small diameter the water-surface is no longer a "free" surface.

Consider a glass capillary tube of radius r submerged at its lower end in water. Owing to the relative magnitude of the three interfacial "free energies" inside the tube, the water-surface assumes the form of a hemispherical meniscus, convex downward. The pressure necessarily drops going downward across this curved surface, by an amount Δp, where $\Delta p = (2S/r)$, S being the "surface-tension" of water. As a consequence of the unbalanced pressures inside the tube (neglecting vertical variations in vapor-density), the water-surface rises until $[p_a + \Delta p - wz$ (inside)$]$ becomes equal to p_a (outside). Then $wz = \Delta p$, or $z = c = (\Delta p/w) = (2S/wr)$. If the datum is taken at the bottom of the vessel, the initial (and final) depth of water in the (large) vessel is h. At elevation $(h + c) = z$, just beneath the meniscus, the pressure is $(p_a - \Delta p) = (p_a - 2S/r)$. The final head at this point is $[(h + c) + (p_a - 2S/r - p_a)/w] = h$. In other words, the head is uniform throughout the body of water; the water is motionless.

It appears that Mr. HORTON has used "head" indiscriminately as being synonymous with "height". In the case of the capillary, the "height" of the meniscus above the free surface (the capillary rise) may be considered a negative pressure head at $z = c$, but the total head (or simply "head") is the same at that point as at any other point in the fluid while at rest. In order to be precise in our nomenclature it would seem well to avoid saying "head" when we mean "difference in head" or "difference in height", and to use the modifiers "pressure", "geodetic", and "velocity" to describe the components of the head. Moreover, in any given problem the head should always be referred to the same datum.

In applying POISEUILLE'S equation to Mr. HORTON'S experiment, we may observe that if the meniscus is drawn up an inclined capillary by suction and allowed to fall, then at a given time when the meniscus is at elevation z the length of fluid-column resisting the flow is $z/\sin a = \ell$ (not unity), where a is the angle of inclination. We assume, as Mr. HORTON did, that the length of the submerged tube is small, and also that r is small enough so that gravity does not distort the meniscus. Then, since the problem is one of uniform flow, $(dz/dt) = v = (-Kh/\ell) = [-Kh(\sin a)/z]$. Expressing the head in terms of elevation and pressure, we get $h = (z - 2S/wr) = (z - c)$, whence $(dz/dt) = -K(1 - c/z)\sin a$. This may be integrated, after separating the variables, giving $K't = z_0 - z - c \log[(z - c)/(z_0 - c)]$, where $z = z_0$ for $t = 0$. By a simple change of variable this reduces to Mr. HORTON'S equation (2), and we thus arrive at the same result without having to introduce additional unnecessary terms.

The reference to the work of BOUSSINESQ given by Mr. HORTON is the same as that given by PORCHET. Apparently there was some misunderstanding on the part of PORCHET as to why BOUSSINESQ introduced the capillary rise into his equations and also as to the application of the equations as thus modified. BOUSSINESQ was concerned primarily with the flow of water in a shallow unconfined aquifer of great extent (relative to its thickness) and having small slopes (both top and bottom). He assumed, as had DUPUIT previously, that the flow-lines were virtually horizontal, that the planes of equal potential were vertical, and that therefore the vertical pressure-distribution was hydrostatic [see reference 2 of the paper, pp. 11-14]. Consequently his equation for nonsteady flow was in the form

$$\mu (dh/dt) = d[K(H + h)dh/dx]/dx = d[K(H + h)d\phi/dx]/dx \qquad (2)$$

with corresponding terms for the y-component of flow. Here μ is the specific yield, K is the transmission-constant, H is the depth below the datum of the base of the aquifer at (x,y), h is ordinate of the free surface at (x,y), and ϕ is the head (la charge) at (x,y).

In taking capillarity into account BOUSSINESQ explained (and this is quoted in part by PORCHET): "A une seconde approximation, il y aurait lieu de tenir compte de la réduction, que j'appellerai ξ, éprouvée par la pression p et par la charge ϕ, dans la nappe, à raison de la tension superficielle de l'eau . . . On aurait donc, non plus $\phi = h$, mais $\phi = (h - \xi)$; et il conviendrait de prendre ϕ, au lieu de h, comme inconnue; ce qui laisserait subsister entièrement, en ϕ, la forme de l'équation indéfinie (2), sauf la substitution, à la fonction donnée H, de la fonction analogue $(H + \xi)$. Car la hauteur $(H + h)$ des volumes prismatiques élémentaires mouillés deviendrait $(H + \xi + \phi)$. . . . Rien ne sera donc changé à la forme des équations du problème ni, par suite, a la solution, sauf toujours la substitution, partout, de $(H + \xi)$ à H, jusque dans la hauteur de la section fluide intérieure, verticale et parallèle à x, fournissant le débit Q de la nappe" [Reference 2, p. 21].

In following BOUSSINESQ'S instructions one would rewrite his equation (2) as

$$\mu (d\phi/dt) = d[K(H + \phi + \xi)d\phi/dx]/dx + . . .$$

Here $(H + \phi)$ is the apparent depth of flow, as might be determined from measurements of the head in observation-wells. In order to take into account the horizontal flow which takes place in the zone of complete "capillary" saturation overlying the water-table it is necessary to augment this depth of flow by the height of capillary rise, ξ, giving the expression enclosed by parentheses. But the capillary rise has no effect upon the horizontal, x-component of velocity, and hence $v_x = -K(d\phi/dx)$, as included in the foregoing equation. The capillary forces, in the aggregate, act vertically upward and therefore have zero, or nearly zero, component in the direction of flow, and this regardless of the texture of the soil. In the case of vertical free-flow, as studied experimentally by PORCHET and by Mr. HORTON, if the head is measured properly one should find that the flow follows DARCY'S law and that for the zone of complete saturation K remains constant--and independent of the boundary-conditions. Failure of PORCHET'S experiments to check with the theory (as seen, for example, by comparing the broken and full curved lines in Mr. HORTON'S Fig. 2) is to be attributed to his assumptions. He assumed the capillary rise to be equal to the supposedly constant reduction in head which began the instant the water-surface entered the sand, and computed it from the change in slope occurring at α on the flow-time diagram. This is perhaps not far from right initially, but capillary rise is a vague term. What is more serious, he neglected the influence of the rate of fall of the liquid surface on the amount, distribution, and subsequent movement of moisture retained by the soil at a given point in the column. As pointed out by Mr. HORTON, PORCHET made no measurements of the head after the liquid surface entered the sand. If he had measured the head-distribution, let us say by means of piezometers spaced vertically along the column, and if he had been able to evaluate the equation of continuity for the entire column, taking into account the variations of moisture-distribution mentioned above, then doubtless he would have been able to revise his theory to fit his experimental results. It is interesting to note that although PORCHET had the right idea in introducing the capillary rise as what might be considered a "safety-factor" in designing systems of parallel horizontal drains, his equations were based on the "DUPUIT assumption" (that is, that the discharge per unit-length of drain is $q = -K \times y\ (dy/dx)$, which is not valid for the high slopes involved. It appears that this is the only problem, of the many he treated, in which the capillary rise was directly involved.

Following a statement to the effect that "numerous series of experiments have shown that flow through a given length ℓ of fine-textured substance . . . is zero unless the hydraulic head exceeds a minimum limit b," Mr. HORTON refers to the work of BALDWIN-WISEMAN. Then, later in the paper, he states in effect: ". . . aquifers supplied by numerous rock-fissures where the size and frequency of the fissures decrease rapidly proceeding downward . . . . may yield no water whatever when the profile has receded into the finer fissures and the slope has decreased to a value less than (b/ℓ), although in the latter case there may still be a definite positive hydraulic gradient or manometric head." This is followed by reference to MILLER-BROWNLIE'S paper. As far as the writer has been able to ascertain, in neither of these papers is there any support of this contention. Perhaps Mr. HORTON has in mind other "series of experiments."

BALDWIN-WISEMAN was concerned with sand-filters. He measured the rate of flow through actual filter-sands and through other, graded sands ranging in particle diameter from 0.0089 inch to 0.165 inch, under various gradients ranging from 0.5 to 40. It is obvious that many, if not most, of his data are for values of REYNOLDS' number above the critical.

In estimating the yield of irrigation wells, MILLER-BROWNLIE resorted to a rather curious device. He states: "It will be observed that I have calculated the cone as having straight sides and have added a few feet for the trumpet-mouth portion of the cone. The influence of the trumpet-mouth is only observed a few hundred feet from the tube, and the remainder of the curve being so nearly flat it may be considered the straight lines of an unrestricted hydraulic gradient. The error thus introduced is slight and the calculations very considerably simplified." Elsewhere he claims that observations in Punjab have shown that the "slopes necessary to cause water-motion have varied from one in 260 in moderately coarse sand to one in 175 in fairly fine sand." Thus it might appear that there had been some experiments. But, he states further: "Observations indicate that any lateral or forward motion of water, where the hydraulic gradients are slightly less than those mentioned, is so slow that <u>for practical purposes it may be neglected</u>, the actual velocity probably <u>not exceeding a few inches per day</u>" [underscoring is mine]. In view of the latter statement it is quite obvious that a minimum hydraulic gradient was merely assumed to simplify calculations.

Referring to Mr. HORTON'S Figure 4, which "shows conditions of zero-flow in a fine-textured aquifer where the constant b is of appreciable magnitude," one might naturally wonder, assuming b is constant (and presumably characteristic of the soil-texture), how the profile ab became established in the first place. According to Figure 4, b varies continuously from a maximum value at the left to a minimum value at the right. It would seem, therefore, that perhaps it is intended rather that there be a constant minimum gradient $(db/d\ell)$, and that equation (3) be written in the form $v = -K[(dh/d\ell) - (db/d\ell)]$. But if that were true, the profile ab should be a straight line.

O. E. MEINZER (U. S. Geological Survey, Washington, D. C.; communicated)--Mr. HORTON reports that in the experiment made by him the water moved in accordance with POISEUILLE'S or DARCY'S law. In his paper as published he also concludes that "transmission-capacities determined from a submerged soil-column are applicable to natural conditions." I am glad to note these conclusions because I believe they are firmly established by the thorough work of many competent investigators. DARCY'S law and its application to natural conditions are fundamentally involved in the quantitative work of the Ground Water Division, as is impressively shown in the recently published treatise on permeability and its field-application by L. K. WENZEL and V. C. FISHEL [W.-S. Paper 887].

After discussing his experiment, Mr. HORTON postulates a constant which he designates b, but he is careful to explain that this constant is not involved in his experiment and he presents no new evidence in support of its existence. He states that it has application to water in extremely fine fissures, soil-interstices, or capillaries. Figure 4 shows hypothetical conditions where b is supposed to be of appreciable magnitude--"where the capillaries are so small that all or most of the water is held as 'bound' water by molecular forces until the hydraulic head exceeds the minimum value of b for the given length of flow." I shall not undertake to discuss the physics of exceedingly fine material, but it appears to me that if all the water is "bound" by molecular forces the material is impermeable and there is no flow through it, whereas if a part of the water is not "bound" that water is in a fluid state and obeys the laws of fluid mechanics, including DARCY'S law.

ROBERT E. HORTON (author's closure; communicated)--The discussion by Mr. JACOB exceeds the original paper in length. The first is in part repetitive and affords a complete confirmation both of the validity of the experiments and the law deduced therefrom by the author governing flow in a capillary tube with a capillary meniscus at one end, while the remainder of Mr. JACOB'S discussion is in part devoted to matters not entered into directly in the author's paper and which the author does not feel compelled either to accept or to answer.

There are some matters which require attention. Anyone who will take the trouble to translate the half-page quotation in French from BOUSSINESQ'S work will find therein nothing which in any way modifies the author's interpretation of BOUSSINESQ'S statement as cited by the author.

The distinction in types of "head" used by the author is neither unnecessary nor confusing. In case of the experiment described, the only head which could be directly measured is the total head. The net head, which governs the flow, can only be determined by deducting the capillary head from the total head [1].

The author is quite familiar with the meaning and physical interpretation of the terms he has used. An effort was made in preparing the paper to present it in language readily intelligible to persons with only a limited knowledge of the underlying physical problems and processes. The author does not wish to belittle in any way the desirability of rigorous presentation of a physical subject. Too often such presentation goes over the heads of those whom it might otherwise interest. The author believes that there is a "happy hunting ground" between the rigorous and usually tedious and pedantic, if not sophomoric, presentation of scientific results, on the one hand, and the still less desirable sketchy, so-called "popular", presentation.

Mr. JACOB'S derivation of the equation of the experimental curve gives the same results as the author's but is neither as simple nor as readily understood.

Mr. JACOB'S statement that capillary forces in the aggregate act vertically upward and therefore have zero or nearly zero effect, is obviously incorrect without limitation. In the author's experiment both gravitation and capillary pull act primarily in vertical directions. Under the conditions of the experiment the horizontal components of both of these forces are reduced in the same proportion when the tube is placed in an inclined position and the relation of the two forces remains unchanged, whether the capillary tube is vertical or inclined at an angle, as in the experiment. It is the difference of these two forces which operates to produce flow in the capillary tube.

Both Mr. JACOB and Dr. MEINZER refer to "DARCY'S" law. DARCY did not discover the law of capillary flow. It was discovered by POISEUILLE, and DARCY, finding that it applied to flow through filter-sands, properly credited it to POISEUILLE. It is primarily a law of physics--not of geology. The author has followed the nearly universal custom of physicists in describing it as "POISEUILLE'S law." Full credit for this law has been given to POISEUILLE, as it should be, in a recent paper by J. F. HERRICK [2].

The purpose of the author's experiment was to determine the effect of capillary tension where free menisci exist in capillaries in connection with certain problems not discussed in the paper. After the experiment was performed the author realized that it served to interpret rationally the results of PORCHET'S experiments. The author is aware of the various lines of work relating to ground-water carried out by PORCHET in addition to these particular experiments but does not feel called upon to discuss these other questions relating to that work which have been raised by Mr. JACOB.

PORCHET platted the rate of flow against time. There is an abrupt break in the experimental curve at the point where the water entered the soil-surface. MEINZER, JACOB, and the author are apparently agreed that this break does not represent a change in the actual transmission-capacity or permeability of the soil. It must, therefore, have been due to a change in the effective hydraulic gradient, and the occurrence of this break is simply and completely explained by the author's experiment.

The real bête noir underlying the discussions of JACOB and MEINZER appears to be the question whether there is a lower limit of direct applicability of POISEUILLE'S law to flow through fine-textured soils resulting from water in the finer pores of the soil being held as bound water at ordinary or small pressures.

Probably the author presumed too much in assuming that this subject would be familiar to those who would read or discuss this paper but regrets that it is wholly impracticable to give more than meager excerpts from the extensive literature on the subject, which is scattered through the literature of physics, hydrodynamics, and physical chemistry relating to the adsorption of liquids on solid boundaries--the non-existence of slip at the boundary in laminar flow, molecular orientation and association, the physics of unavailable soil-moisture, and the rheology of plastic and non-Newtonian flow. All this literature has a direct bearing on the question of what happens in case of extremely fine capillaries in the soil in which the water is bound at low heads or pressures but may become free and flow at higher heads or pressures.

Water apparently is a nearly perfect Newtonian fluid under ordinary conditions but becomes a non-Newtonian fluid close to a boundary-wall. This change is apparently due to a change of structure involving orientation of molecules and increased molecular association. These conditions have suggested that unfree water is of the nature of some form of ice [3].

The constant b, equation (3) in the author's paper, is the well-known yield-value for plastic flow.

The constant c is the height of capillary rise, and there is a corresponding constant where bound water plays a role in capillary flow, under conditions such as those in the author's experiment. As pointed out in the author's paper, b and c are not necessarily of the same physical nature. While in the case considered in the author's experiment, c actually is a constant, in a natural aquifer with pores of varying sizes from coarse to extremely fine, c is quite certainly not a constant. It has one value or may be negligible for the larger pores and has increasingly higher values as the fineness of the pores increases. Under these conditions the flow instead of being of identically the same nature as plastic flow, with a definite yield-value, as indicated by comparison of equations (3) and (4), would be more nearly of the nature of the flow of a non-Newtonian fluid [4]. Obviously it is difficult to experiment directly with flow of water at distances from the boundary-wall as minute as those involved where bound water is concerned. The existence of finite yield-values in fluids has, however, been determined experimentally in salt solutions by BLAIR and SCOFIELD [5, 6]. The relations of fluid-flow, pseudo-plastic flow or flow of a non-Newtonian liquid, and true plastic flow, seem first to have been pointed out by BINGHAM who has given [7, pp. 217 and 323] an equation similar in form to the author's equation (3). Later revisions of BINGHAM'S equations retain the same form.

Conditions of flow in a soil with very fine capillaries, under extremely slight pressures, are, however, further complicated by the usual presence of colloidal material with an extremely high water capacity--in fact, it is fair to say that no adequate, complete theory governing all the phenomena of flow close to a boundary-wall under such conditions has yet been formulated.

In a paper published in these Transactions [9], CHARLES H. LEE states:

> "Laboratory tests show that dense clays are absolutely impermeable under the hydraulic pressures ordinarily occurring in nature. This condition results from the compact arrangement of particles, interstitial spaces being so small that the forces of molecular attraction extend across them. Under such a condition, the forces of gravity and capillarity cease to operate and interstitial water becomes solidified."

LEE points out that such a soil may under suitable conditions be "slightly permeable because of numerous interstitial spaces which exceed the range of molecular attraction." This seems to be precisely the case referred to by MILLER-BROWNLIE [18].

B. A. KEEN [8] devotes an entire chapter to plastic flow and points out the identity of form of the equation therefor with the POISEUILLE law for flow through a capillary with a free meniscus at one end.

NORTHROP [10], discussing POISEUILLE'S law, states:

> "The simplest test of this law is the fact that the rate of flow is directly proportional to the pressure. With truly viscous liquids this is true over a wide range, provided the capillary is small. With low pressures it is frequently found, especially in the case of colloidal solutions,

such as gelatin [cf. BOGUE, 11], that the rate of flow decreases more rapidly than the pressure. This is an indication that the substance is plastic rather than viscous, that is, it requires definite force to cause a deformation of the mass. This is very apt to be the case in colloidal solutions due, presumably, to the existence of a definite structure [cf. McBAIN, 12]."

What happens in case of flow through a matrix of an aquifer containing pore-sizes ranging from large to extremely minute, under sufficiently low gradients, is that at larger gradients POISEUILLE'S law in its simpler form, as commonly used, gives highly exact results, for the reason that the percentage of pores in which bound water then exists is extremely minute. As the pressure-gradient becomes less and less, a larger proportion of the pores contain bound water, with resulting departures from the simple form of POISEUILLE'S law.

Finally, if the soil-texture is sufficiently fine, a minimum gradient is reached for which all of the pores contain bound water, and flow ceases. This may not be of much practical importance in case of most natural aquifers. It is, however, of considerable scientific interest and the underlying principles are of great practical importance in connection with certain other hydrologic phenomena which need not be discussed here.

STEARNS' excellent experiments on Fort Caswell (North Carolina) sand [W.-S. Paper 596-C, U. S. Geol. Sur.] afford accurate confirmation of POISEUILLE'S law as applied to this soil for larger heads but the same experiments for extremely small heads (two mm or less) on the length of sample considered show consistently larger departures from POISEUILLE'S law as the head decreases. For a head of 0.5 mm the actual flow-rate as shown by five consistent experimental values is little more than 50 per cent of that given by POISEUILLE'S law. As the head increases, the observed flow approaches that given by POISEUILLE'S law and the experiments for extremely low heads are well represented by a curve of precisely the type required for pseudo-plastic flow. These departures from POISEUILLE'S law do not appear to be capable of quantitative explanation as the result either of evaporation or variation of temperature.

Mr. JACOB has raised the question of MILLER-BROWNLIE'S observation regarding cessation of flow into wells in India in certain soils, under small gradients, and has called attention to experiments of MEINZER [16] and FISHEL [17] purporting to prove that this condition could not occur. Discussion of this question would be unprofitable here for the reason that, strangely enough, neither MILLER-BROWNLIE nor MEINZER and FISHEL give any specific information as to the fineness of the soil they are considering. From the preceding discussion it will be seen that whether flow will cease altogether or cease to follow the POISEUILLE law as ordinarily applied, depends entirely on the fineness of the finer pores in the matrix of the soil or aquifer.

To save space the author purposely cited only the latest of several papers of BALDWIN-WISEMAN of interest in this connection. The paper cited [3] contains [pp. 38-39] references to the earlier papers [14, 15]. It also states [p. 38]:

> "The flow-conditions vary directly as the ratio h to L but at a slightly different rate, the variation being more marked at low pressures than at high pressures."

The experiments of BALDWIN-WISEMAN to which the author intended to call attention were those on flow through consolidated materials such as sandstones and oölites, not, as assumed by Mr. JACOB, the experiments on flow through filter-sands. The experiments on flow through rocks and consolidated materials are given in references [14] and [15], and platted directly as taken, they certainly do not confirm POISEUILLE'S law. It is interesting to note, however, that BALDWIN-WISEMAN found and definitely measured compression of the softer rock-samples (usually six inches thick) by water-pressure as low as five pounds per square inch. This pressure reduced the porosity, especially near the surface of the sample, and this must be taken into account in interpreting these experiments.

Finally, POISEUILLE'S law appears to be fully confirmed for practical purposes within a wide range of pressures and fineness of soil-pores. It is, however, subject to limitations in both directions, the upper limit occurring when the gradient and velocity become sufficiently great so that the flow in the larger pores is turbulent. The lower limit occurs when the gradient is so slight that water in the finer pores is no longer mobile but is held by molecular or capillary forces. The change of law of flow is not abrupt in either case in a soil or matrix containing pores of various sizes.

Engineers, physicists, and soil-physicists alike are generally familiar with the fact that water held in minute capillaries cannot be removed by gravity alone under ordinary natural conditions. The mere fact that these fine capillaries happen to be within an aquifer does not modify this.

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WATER-STORAGE BY CALIFORNIA HILL SOILS IN THE SEASON OF EXCESSIVE RAINFALL, 1940-41

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Introduction

The hydrology of the excessive-rainfall season of 1940-41 in southern California is unusual. The season was marked by high-total rainfall and long-continued storms of moderate intensity. In view of the amount and distribution of the rains, especially in the peak-period in February and March, wide-spread, although not necessarily intense, floods were expected. That they did not materialize, except locally, is evidence of the large quantity of precipitation absorbed by soil. By some, this large volume of rain-water absorption is deemed to have resulted chiefly from the fact that, under the conditions of moderate-rainfall intensities, the soil, irrespective of the nature of the surface and of the type of vegetal cover, was able to take water in fast enough to reduce or prevent flood-producing runoff. Data presented here support an alternate hypothesis, namely, that reduction and retardation of storm-runoff was accomplished through the influence of plant-cover.

The data presented in this paper are from hill-culture studies made primarily to determine economic erosion-control measures for improvement of land-use in the lima-bean erosion-problem area of California, an area also known as the southern California fog-belt. The source of the data is the hill-culture field-station situated a short distance south of Capistrano on steeply rolling hills near the ocean.

Data

Meteorology--Insert A in Figure 1 shows monthly distribution of rainfall, 1940-41, at Capistrano in comparison with the long-time average monthly distribution at Los Angeles. At most stations in southern California, total precipitation for the year 1940-41 exceeded the normal by 100 per cent, or more. For example, the average annual rainfall at Los Angeles is about 15-1/2 inches; for 1940-41, the total was 32-3/4 inches.