A New Evaporation Formula Developed

Empirical Statement Based on Physical Laws Agrees with Observed Facts and Is Held To Be an Improvement Over Existing Formulas

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A new and logically developed empirical formula for evaporation from bodies of water is herein presented in confidence that hydraulic engineers generally will find it a better tool than any heretofore provided for studying evaporation. Most attempts to find evaporation at a given time and place have been confined to observations of evaporation-pan losses and inadequate formula computations. It need not be argued that evaporation needs to be better considered in many of our hydraulic investigations; we have only to realize that in summer a major part of the rainfall never reaches the ocean—disappearing into the atmosphere in a comparatively short time after precipitation.

Physics of Evaporation

Starting, as a working basis, with the fundamental principle of the "difference theory of evaporation," which is that the observed rate of evaporation or condensation from any liquid surface represents the difference in the rate of vapor emission and vapor return to the liquid surface, an effort has been made to follow out this theory to its logical consequence in accordance with the observed facts and physical principles, especially with reference to the kinetic theory of gases.

A theory or formula for evaporation should be in accordance with the following experimental facts:

1. Equilibrium or zero evaporation occurs in still air when the vapor pressure, here denoted as $V$, corresponding to the liquid surface temperature $T$, equals the vapor pressure, $v$, existing in the air.
2. Evaporation continues in saturated air when $V > V_a$ where $V_a$ is the maximum vapor pressure at air temperature $T$.
3. When $v > V$, condensation occurs in still air.
4. Wind increases evaporation and reduces condensation.

The maximum effect of wind on evaporation occurs for wind velocities of 15 to 20 miles per hour, beyond which further increase in the wind velocity does not appreciably increase evaporation rate.

6. Ebulition is a special case of evaporation where $v = V$ equals total pressure on the liquid surface, the air adjacent to the liquid being displaced by vapor and vapor removal accomplished by convection or mechanical action.

The principal processes involved in evaporation and condensation are:

1. Vapor Emission—The rate of emission depends on the nature of the liquid, the temperature of the liquid surface and the external or barometric pressure thereon, and is not appreciably affected by wind velocity.
2. Vapor Removal—This is accomplished in nature by three different processes: (a) by diffusion; (b) by convection; (c) by wind or mechanical action. In the presence of either convection or wind action, which are much more rapid processes, diffusion becomes in effect inoperative, since the vapor is removed before diffusion can produce any appreciable result. Convection, like diffusion, is probably negligible when there is relatively high wind velocity, but presents a case of considerable interest when evaporation takes place into sensibly still air.
3. Vapor Return—This is closely related to the two preceding processes. The rate of vapor return to the liquid surface is dependent primarily on the vapor tension over the liquid surface and is therefore controlled (a) by the vapor tension in the air at a little distance from the liquid, and (b) by the nature and activity of the processes of vapor removal.

Old Dalton Formula Inadequate

John Dalton, in 1802, found experimentally that the rate of evaporation in still air is in accordance with the formula:

$$E = C'(1 - v)$$

where $C'$ is a constant. It does not appear that Dalton attempted to deduce a formula including a factor for wind correction as such. However, he found that a strong wind made the amount of evaporation double that taking place in still air. He concluded that the increase in evaporation rate was proportional to the wind velocity and included in his tables corrections to be made for a moderate breeze and a brisk wind.

Following Dalton's suggestion, other investigators assumed a form of expression to represent evaporation in which the wind-correction factor, usually in the form $(1 + K v)$, almost invariably appears as a factor by which the total measured evaporation rate in still air is to be multiplied, $v$ being the wind velocity and $K$ a constant.

In accordance with the Dalton formula, with the form of wind factor hitherto commonly used, the rate of evaporation increases indefinitely as the wind velocity is increased. This is obviously incorrect, since the rate of evaporation cannot in any event exceed the rate of vapor emission, and the latter is not affected by wind velocity in the absence of waves and spray. There must be for each water-surface temperature a maximum rate of evaporation, which rate cannot be increased by further increase in the wind velocity.

For the conditions in which ebulition ordinarily occurs, $v = V$ just above the liquid surface, and the Dalton formula fails in this case, since it makes $E = 0$ for these conditions.

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Some of the most elusive phenomena of nature are those involving overlapping and alternate causes such that the phenomenon may be the result of one cause under certain conditions and of other causes under other conditions, or of two or three causes cooperating. Sometimes the causes are wholly or in part mutually exclusive. The removal of emitted water vapor from a water surface is a case of combined causality. Convection and wind each operates practically to the exclusion of diffusion. Relative high wind is probably exclusive of convection also; but wind and convection may cooperate when the conditions favor the latter and the former is slight. These conditions make the determination of the exact expressions for wind and convection effects a difficult matter.

Wind and Convection Introduced

The removal of vapor from a liquid surface by wind action corresponds to a condition of natural exhaustion to wind velocity and \( \theta > \eta_a \), the convection factor may be added to the measured wind velocity. Under ordinary conditions occurring in nature, except the case where warm days are followed by cool nights, convective vapor removal may be relatively unimportant; but it becomes important under artificial conditions where the water temperature approaches the boiling point.

Rim Factor

For the purpose of comparing the calculated with the observed evaporation from evaporometer pans, the wind factor should be further modified to allow for the reduction in wind effect by the projecting rim of the pan. This correction may be made by subtracting from the measured wind velocity a quantity \( p \), the value of which is, approximately,

\[
p = \frac{10d}{D}
\]

where \( d \) = depth of water in the evaporometer below the rim of the pan, in inches, and \( D \) = diameter of evaporometer in inches.

A Working Formula for Evaporation

Let

\[
\psi = [2 - e^{-kW} - \phi]
\]

then

\[
E = C[\psi V - \phi]
\]

The value of the wind coefficient, \( k \), is about 0.2. Values of the function \( \psi \) may be read directly from the accompanying diagram, by the aid of which, in conjunction with formula (3), the calculation of evaporation becomes very simple. If there is no wind or convection \( \psi = 1 \), and equation (3) becomes identical with Dalton's formula for evaporation in still air, in which case vapor removal takes place by diffusion.

Strictly, this formula applies only to a small element of area \( dA \), for which \( C = 0.40 \), where \( E \) is the evaporation depth in inches per 24 hours. For larger areas or evaporator pans, an area factor must be introduced as hereafter described, but provisionally the value of \( C \) may be taken at 0.36 for a pan 12 in. square. The numerical factors and coefficients given are subject to revision, since with existing experimental data it is in general impossible to wholly separate the convection, rim and wind effects from the area factor.

The value of the function \( \psi \) varies from unity for evaporation into still air to 2.0 under maximum wind or convection effect. It follows that according to this formula, the maximum effect of wind is to double the evaporation rate into perfectly dry air. If, however, the air is humid, the evaporation rate when subject to wind action may be several times as great as into still air having the same degree of humidity.

For ebullition, \( \phi = 2 \), and \( r = V \), and the evaporation rate according to formula (1) per unit of evaporation surface, whether external to or contained within the liquid, becomes proportional to \( CV \).

Condensation occurs only when \( r > \phi V \). Thus, according to the new formula, conditions may be such that condensation would take place in still air; but if there is wind, there may be a slight evaporation instead. If, however, the humidity in the air is sufficiently large so that
v > 2V, condensation would occur in the presence of wind however great its velocity. Thus wind would tend to increase evaporation, but would tend to decrease condensation, whereas in accordance with most evaporation formulas hitherto presented the effect of wind would increase both evaporation and condensation.

The new formula is in accordance with experience, which shows that the conditions under which condensation occurs in nature are greatly restricted as compared with the conditions permitting of evaporation. Condensation or dew rarely occurs on windy nights. As a further test, experiments were made to determine the effect of wind on the condensation of moisture on the surface of cans containing ice and water, and mixtures of ice and salt, respectively. It was found under the given humidity and temperature conditions that whereas condensation took place freely on both cans in still air, the condensed moisture was evaporated from the can containing ice and water by a strong wind; but wind action was not adequate to prevent condensation on the can containing a mixture of ice and salt.

**Areal Factor Applied**

Experience has shown that the evaporation rate from large pans is less than from smaller pans subject to the same external conditions and with the same water temperature. Engineers are generally of the opinion that the evaporation rate from a broad lake or other water surface is less than that from a small evaporometer pan exposed at the margin of the lake, the area factor or average rate being variously estimated at from 50 to 75%. If it is assumed that a fraction, m, of the vapor evaporated from an element of area near the windward shore of a water surface is carried horizontally by wind action into the vapor blanket, as it may be called, lying immediately over the next element of surface to leeward, then it is evident that the evaporation rate should progressively decrease proceeding to leeward over a broad water surface, until the air vapor tension v reaches its maximum value for the given conditions.

It is well known that under conditions favoring evaporation on the shore evaporation still continues even at great distances from the shore over a smooth water surface in a moderate breeze. The vapor blanket referred to undoubtedly exists, however, and materially reduces the evaporation over lakes as compared with shore conditions. The thickness of the vapor blanket in the sense here used is certainly slight, so that vapor tensions or humidities as ordinarily measured, even over water surfaces, are not affected by it.

Developing the theory of the evaporation rate over broad water surfaces in accordance with formula (1), and on the assumption that a fraction, m, of the vapor is carried along the water surface by wind action, it is found that under conditions such that active evaporation occurs near the shore the evaporation rate decreases rapidly proceeding to leeward, becoming constant at some distance x from the windward edge of the water surface.

\[ x = \frac{1}{m} \left( \frac{\psi V - v_o}{\psi V - \nu_e} \right) = \frac{1}{m} \left( \frac{E_o}{E_e} \right) \]

(4)

where \( v_0 \) and \( \nu_e \) are the vapor pressures in air at windward edge and to the leeward of \( x \), respectively, the latter determined as hereafter described: \( E_o \) is the evaporation rate at water's edge determined by formula (1), and \( E_e \) is the evaporation rate (constant with respect to \( x \)) to the leeward of \( x \).

\[ E_e = C[\psi V - \nu_e] \]

(5)

For conditions such that evaporation occurs at the shore—that is, for \( \psi V > \nu_e \)—make \( \nu_e = \psi V \) and

\[ E_e = C[\psi V - \psi V] = 0 \]

(6)

For conditions such that there may be either evaporation or condensation depending on the relative humidity—that is, for \( \nu_e > \psi V \)—make \( \nu_e = \psi V \) and

\[ E_e = C[\psi V - \psi V] = 0 \]

(7)

In other words, the evaporation or condensation rate in this case is either zero or proceeds at a gradually decreasing rate to leeward, over the whole water surface, approaching zero as a limit. In this case \( x \) is infinite. For ordinary evaporation conditions—that is, \( \psi V > \nu_e \)—the distance \( x \) is usually small and finite, generally ranging from a few inches to a few feet, so that for a large water surface the evaporation will proceed at the same rate over practically the entire water surface, the rate being

\[ E_e = C[\psi V - \nu_e] \]

The distance \( x \) depends on the fraction \( m \) of vapor carried horizontally forward. The values of \( m \) have not been determined by direct experiment. They may, however, be inferred from the results and experiments on the relative evaporation losses from experimental pans of different diameters. The factor \( m \) would be unity for a perfectly uniform horizontal stream-line motion of the vapor. Its average actual value appears to be about 0.5, ranging from as low as 0.6 or 0.7 for steady winds to 0.3 to 0.4 for gusty winds and becoming approximately zero over water surfaces broken by waves and over rough land surfaces or vegetation.

The constant evaporation rate to the leeward of the point \( x \) is independent of the factor \( m \).

The case of an evaporation pan on shore, if \( x \) is less than the diameter of the pan, then under ordinary evaporation conditions the evaporation rate decreases rapidly from some value \( E_o \) at the windward edge of the pan to a value \( E_e \) at the point \( x \) somewhere over the surface of the pan, while to the leeward of \( x \) the rate remains constant.

From water's edge to the point \( x \), the evaporation rate at any point \( x \) is

\[ E_x = E_o e^{-mx/\psi V} \]

The average evaporation rate over a strip of unit width and length \( x \) in the direction of the wind is

\[ \frac{E_{av}}{mC_x} \left( 1 - e^{-mx/\psi V} \right) \]

(8)

**Use of Evaporation Pans**

Since the distance \( x \) is apparently relatively small in most cases, it appears that a floating atmometer exposed either near the leeward shore or at a distance of a few rods from the windward shore of a lake should give approximately the true evaporation rate over nearly the whole water surface.

The land-exposed evaporation pan appears to be about the poorest device humanly contrivable for the purpose of determining the evaporation losses from broad water surfaces. It is evident from these considerations that agreement between the results obtained from land-exposed evaporimeters with different exposures or of different sizes.
is not to be expected. Two evaporimeters of different sizes may give nearly identical results under certain conditions, but may be widely apart under other conditions, depending on the position of the point \( x_e \) over their surfaces.

Probably the results of floating atmometers exposed at some little distance from the shore of a lake can safely be used for estimating evaporation from broad water surfaces. Even these results generally require correction for rim effect, and for difference of temperature of the water within and outside the pan.

**Calculating Evaporation**

Owing to the practical difficulties of so measuring evaporation as to determine the true loss from a broad water surface, it appears probable that where suitable data are available, calculation will be found more satisfactory than attempts at direct measurements of evaporation losses. Most of the data required for calculating evaporation are contained in the annual summaries of the meteorologic conditions at regular United States Weather Bureau stations. In calculating the evaporation rate over a broad water surface, formula (5) is to be used instead of formula (1), which applies to shore conditions only. For example: Required the evaporation rate over a lake for the following conditions:

\[
\theta = 70^\circ = \text{water temperature}; \\
V = 0.732 \text{ in. of mercury}; \\
\nu = 60^\circ = \text{air temperature}; \\
k = 60\% \text{ relative humidity.}
\]

Then

\[
V_a = 1.022 \text{ in.;} \\
\nu = 0.6132 \text{ in.;} \\
\psi = 5 \text{ miles per hour wind velocity;} \\
\phi = 1.61; \\
\phi V = 1.200;
\]

and since \( \phi V > V_a \)

\[
V_e = V_a = 1.022.
\]

At the water’s edge

\[
E_0 = 0.40[1.200 - 0.6132] = 0.2348 \text{ in. per 24 hours.}
\]

Beyond \( x_e \),

\[
E_e = 0.10[1.200 - 1.022] = 0.0712 \text{ in. per 24 hours.}
\]

Ratio \( \frac{E_0}{E_e} = 3.3. \) If \( m \) is taken at 0.5, then

\[
x_e = \frac{1}{0.5 \times 0.40} \log_{10} 3.3 = 5.97 \text{ ft.}
\]

This represents ordinary summer afternoon conditions. Actually, the evaporation loss from the lake would be greater than 0.0712 in. because of the increased evaporation rate over the narrow margin to windward of \( x_e \) because of higher water temperature on shallow shore margins all around a lake, and because winds are never steady, and a gusty wind or the presence of waves would tend to break up the vapor blanket, prevent stream-line flow of vapor, and cause the evaporation rate to approach at times the upper limit, 0.2348 inch.

For night conditions, with water warmer than the air, we would have, say

\[
\theta = 65^\circ; \\
V = 0.6132 \text{ in. of mercury;} \\
\nu = 60^\circ; \\
V_a = 0.5170;
\]

\[
k = 0.80; \\
\nu = 0.4136; \\
w = 2.
\]

But there is convection, so that in effect

\[
w = 2 + \sqrt{65^\circ - 60^\circ} = 4.25; \\
\phi = 1.45; \\
\phi V = 0.8932.
\]

Again, \( \phi V > V_a \) and \( \psi = V_a \); \( E_0 = 0.1918 \) and \( E_e = 0.1500 \). The average ratio \( \frac{E_0}{E_e} \) for both day and night is 2.30, or the indicated evaporation over the lake is 43.4% of the shore rate. The reason for the higher night value of \( E_e \) lies in the reduced air temperature and capacity of space for vapor, hence reducing the blanketin effect. Some of the vapor emitted under these conditions is condensed a little above the water surface and carried away mechanically. Bigelow found that evaporation proceeds at night much more rapidly than is commonly believed, or than the Dalton formula would indicate.

In calculating evaporation it is always necessary to consider separately day and night conditions. Daytime temperatures may be taken as approximately one-third of the mean of the 8 a.m. maximum and 8 p.m. values, while the mean night temperature is approximately one-third of the mean of the 8 p.m. minimum and 8 a.m. values. The day wind velocity is commonly about \( \frac{3}{4} \) to \( \frac{1}{2} \) the 24-hour mean, and the night wind velocity is from \( \frac{1}{2} \) to \( \frac{1}{4} \) the mean.

Difficulties hitherto experienced in reconciling observed evaporation with computed amounts as determined by the Dalton or other formulas have apparently arisen from three causes: (1) Improper application of the wind correction factor; (2) neglect of convection; (3) failure to recognize the fact that the area factor is not constant for a given size of pan or evaporating surface, but that the ratio of the average evaporation rate over such a surface to the evaporation rate at the windward edge varies with each change of meteorologic conditions.

**American Cement Mill Near Guatemala City**

A new cement mill with a capacity of from 50,000 to 100,000 bbl. of cement per year has just been opened about one mile from Guatemala City by American interests acting under a special concession of the Guatemalan Government, according to a recent consular report from Consul S. T. Root, of Guatemala City. The plant has been under construction for two years and involves not only the mill itself, but two miles of railway connecting with the port. Practically all the machinery was manufactured in the United States. The new company is restricted under its franchise to a maximum charge of $4 (United States gold) per bbl. for its product. The managers state that they will be able to manufacture cement at a cost of $1.25 per bbl. and that it can be sold much below $4 per bbl. and still produce an adequate return on the capital. Before the European War most of the cement in Guatemala was brought from Belgium and Holland, and American manufacturers could hardly meet the competition. Since the war, practically all the cement importation has been from the United States, but transportation charges brought the cost in Guatemala City up to as high as $7.45 a bbl.