11. PARTIAL ANALYSIS APPLIED TO SCALE PROBLEMS IN SURFACE MOISTURE FLUXES

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Abstract. Partial analysis is applied to the problem of predicting the moisture fluxes of infiltration and evaporation at land surfaces. The discussion covers the widely different scales of the soil particle, a soil pedon, a field, a basin and a biome. It is suggested that simplified models can be used at these different scales to provide bounding solutions to the integrated behaviour of land surface fluxes of interest in linking hydrologic models and general circulation climate models.

1. Background to Problem

1.1. Fluxes at Land Surface

The treatment of land surface fluxes in general circulation models has been identified as one of the key areas where improved simulation is required in climate studies. The land surface components of the classical general circulation models have been reviewed by Carson (1982) and some new approaches described by Dickinson (1984), Sellers et al. (1986), Abramopoulos et al. (1988) and Entbekabi and Eagleson (1989). If our knowledge of hydrologic processes is to be applied at the scale of a general circulation model, then a very high degree of parameterization from the micro-scale to the macro-scale will be required (Dooge, 1982). Since the basic equations are non-linear, this scaling problem poses serious difficulties. The aim of the study presented in this paper is to seek methods of expediting the solution of that problem through various methods based on the approach known as fractional analysis or partial analysis.

The linking of a climate model and a hydrologic model involved fluxes at the land surface in moisture, energy and momentum as indicated in Figure 1 (Eagleson, 1982a) and linkages between these fluxes. In this paper we will be concentrating on the moisture fluxes of evaporation and infiltration but must keep in mind that the state of the soil moisture zone will also affect the other two types of flux. Even when evaporation and infiltration are atmosphere-controlled rather than soil-controlled, the heat fluxes are affected by any variations in moisture flux since the net energy available is split between latent heat of evaporation and sensible heat. Even the energy available at the surface is affected by the state of the soil surface since the albedo of the surface varies with the moisture content. Figure 2 due to O’Kane and based on a discussion by Rowntree (1984) shows some of the complex feedback that result from changes in soil moisture content.

Partial analysis seeks either (a) to reduce the problem to component parts by breaking the weaker feedback loops or else (b) to obtain simple macro-relationships.
that simulate reasonably well the overall effect of those interlocking micro-processes which are themselves too complex to be modelled in full detail. The alternation of wet periods and dry periods gives rise to four basic types of moisture exchange between soil and atmosphere. Atmosphere controlled infiltration will only occur in the early stages of a rainstorm. Once surface saturation occurs, the rate of infiltration is determined by the soil characteristics and condition rather than the rate of rainfall intensity. After the rainfall ceases, a wet soil will be capable of evaporation at the potential rate determined by atmospheric conditions. During this potential phase of first stage drying there will be a continual decrease of the moisture content at the surface. As the lower layers of the soil also dry out, delivery of water to the surface becomes more difficult and second stage drying takes place with evaporation less than the potential rate. The succession of these four types of moisture flux at the surface is shown diagrammatically on Figure 3, starting at the right and moving clockwise through the cycle. At the start of wetting a dry soil or
of drying a wet soil, the interchange of moisture between atmosphere and land surface is controlled by atmospheric conditions and consequently in any simulation the atmospheric component is unaffected by the hydrologic component. Following either surface ponding during a wet period of surface desiccation during a dry period, the surface flux becomes soil-controlled and hence the hydrologic component directly affects the atmospheric component. From the point of view of the hydrologic model, a boundary condition applies in the atmosphere controlled stage. The input to the model, i.e. rainfall or actual evapotranspiration, varies with time and computations of the hydrologic model must be performed on-line. In the second soil controlled stage, the surface moisture content is known and provides the boundary condition. As this prescribed value is constant, the computations of the hydrologic components can be performed off-line in this stage.

1.2. METHODS OF PARTIAL ANALYSIS

The guiding principle suggested by Hamming (1962):

"The purpose of computing is insight not numbers".
SOIL DRYING

atmosphere controlled

Upward Flux

Ea = Ep

Ea < Ep

G < R

G = R

Downward Flux

soil controlled

Fig. 3. Switching of surface flux control.

SOIL WETTING

Ep: Potential evaporation
Ea: Actual evaporation
R: Rainfall rate
Q: Infiltration rate

t_d: time to surface desaturation
t_E: total duration of drying
t_p: time to surface ponding

The neglect of the latter principle can also be an impediment to progress. The suggestion of Polya (1957), that the key to solving a complex problem may be to solve a simpler problem first and then return with renewed insight to the complex problem, is relevant in this connection.

There is a wide variety of techniques in partial analysis which enable us to gain insight into the nature of a solution which cannot be obtained in explicit form. Kline (1965) describes as fractional analysis (i.e. partial analysis):
Partial analysis includes the various forms of dimensional analysis, the absorption of parameters, the transformation of both independent and dependent variables, followed by the derivation of simplified solutions of the resulting canonical equation that give either close approximations or limiting forms of the complete solution. The most common forms of such simplified solutions are similarity solutions, linearized solutions, perturbation solutions, non-linear solutions of simplified equations, and multiple domain solutions.

Partial analysis can in many instances provide plausible bounds to the solution of complex problems whose solution is unknown or is itself so complex that the nature of the solution is obscured by the weight of algebraic expression involved. Examples of the success of such partial analysis can be cited from many disciplines. Partial analysis can suggest ways in which complex problems can be sub-divided using reasonable but different assumptions in the separate components as in boundary layer theory. Partial analysis can evaluate the sensitivity of different types of output to simplifying assumptions. Thus hydrologists are largely interested in storage volumes and boundary fluxes in the unsaturated zone, rather than soil moisture profiles. Hence simplified solutions that give realistic storage volumes and boundary fluxes may be adequate for many purposes including the treatment of land surfaces in general circulation models. Partial analysis is relevant in both deterministic and stochastic methods. In the stochastic approach, simple zero-order models have been used in climate studies to gain insight into atmosphere – ocean interaction (Hasselman, 1976) and into atmosphere – land surface interaction (Delworth and Manabe, 1988). The concern in the present paper is with the application of the general approach of partial analysis to moisture fluxes at land surfaces in the form of infiltration and evaporation. In particular, the approach will be used to examine the relationship between models of the occurrence and movement of soil water at the widely different scales of a soil particle, a pedon with uniform properties, a field in which spatial variation of these properties occurs, a catchment basin, and a regional biome.

1.3. SCALES IN SOIL MOISTURE ANALYSIS

The problem of the parametrization of micro-scale processes in a macro-scale model is a major stumbling block in many scientific disciplines. Within hydrology itself, the problem takes many forms (Dooge, 1986). A necessary preliminary to any discussion of this problem is the clear characterisation of a limited number of spatial scales at which an attempt will be made to formulate relationships of diagnostic and prognostic value. The factors that influence such a choice are the scale at which existing theoretical or simulation models are constructed, the scale at which available data likely to be useful for validation and calibration has been
assembled, and preconceptions about the scales at which new uniformities of behaviour might be identified.

In the present context, the two scales of particular interest are the scale of a HAPEX field experiment which is just below a length scale of $10^5$ m and of GCM grid where length is just above this figure. Since the problem is a complex one, it is desirable to attempt the formulation of relationships both at a scale an order of magnitude below this figure and at a scale an order of magnitude above it. In this paper we will refer to the upper scale of about $10^6$ m as the biome scale in order to emphasise the importance of vegetation in the promotion of equilibrium at this scale. The lower scale of about $10^4$ m will be referred to as the basin scale since the tendency towards equilibrium at this scale depends on the morphology of the basin and the interaction between the differing modules of slopes, channels and aquifers that form the basis of the model formulation at the basin scale. Three smaller scales will also be discussed. These are (a) a field scale of about $10^2$ m in which there will be variations of local parameters but no difference in the growing equations; (b) a pedon scale of about $10^{-2}$ m at which the one-dimensional form of Darcy's law is assumed to operate without spatial variability and (c) a particle scale of $10^{-3}$ to $10^{-6}$ m at which physical laws based on viscosity and surface tension can be applied. These five scales are illustrated on Table I together with the principle variables and parameters used in the formulation of soil moisture relationships at each scale. Though there are some differences in nomenclature, these five scales can be readily related to the range of scales used in discussions of the same general problem by Bonnet (1982), Dooge (1982), Ganoulis (1986), Wood et al. (1988a).

<table>
<thead>
<tr>
<th>Scale</th>
<th>Variables</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle</td>
<td>$\theta, h, k$</td>
<td>$a, \sigma, \cos \phi, \mu$, shape, packing, size</td>
</tr>
<tr>
<td>$(10^{-3} - 10^{-6} \text{ m})$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pedon</td>
<td>$t_p, f(t), f(t)$, $t_d, e(t)$</td>
<td>$K(\theta), D(\theta)$, void ratio</td>
</tr>
<tr>
<td>$(10^{-2} \text{ m})$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td>$f(t), e(t)$</td>
<td>$S, f_{\text{ult}}$</td>
</tr>
<tr>
<td>$(10^2 \text{ m})$</td>
<td></td>
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</tr>
<tr>
<td>Basin</td>
<td>$f_p(a), e_p(a)$</td>
<td>$S(a), f_{\text{ult}}(a)$</td>
</tr>
<tr>
<td>$(10^4 \text{ m})$</td>
<td></td>
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</tr>
<tr>
<td>Biome</td>
<td>$E(t)$</td>
<td>climate, soil, vegetation</td>
</tr>
<tr>
<td>$(10^6 \text{ m})$</td>
<td></td>
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</table>
2. Partial Analysis at Particle and Pedon Scale

2.1. Physics of Porous Media Flow

It is worthwhile reviewing briefly the main features of the occurrence and movement of soil moisture at the particle scale in order to provide a background for the study of spatial variability at the field scale and at higher orders of parameterisation. These topics are dealt with in more detail in Kuhnel et al. (1990). Water can only be held against gravity in the unsaturated zone because of the capillary forces arising from the curvature of the air-water interface in accordance with the basic relationship (Childs, 1969, Chapter 8):

$$p_a - p_w = \sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right),$$

(1)

where $p_a$ is the pressure in the soil air, $p_w$ the pressure in the soil water, $\sigma$ the surface tension of water, $r_1$ and $r_2$ the principle radii of curvature at the air-water interface. For simple geometries, the relationship between the radii of curvature and moisture content can be derived analytically (Haines, 1925, 1930; Collins, 1961). Such cases of ideal soils can be used to gain insight into the influence of various soil characteristics (average particle size, nature of packing, shape of particles, etc.) on the soil moisture characteristic curves of real soils. The variation between the values for the range of possible packings is less than one order of magnitude which is small compared with the variation between the average particle size of different soils which may amount to several orders of magnitude.

Darcy's law postulating that the face velocity in a porous medium is a linear function of the potential gradient was based on experiments involving one-dimensional saturated flow (Darcy, 1856). Nearly 100 years elapsed before the applicability of Darcy's law to unsaturated flow was verified experimentally (Childs and Collis-George, 1950). Partial analysis of unsaturated flow suggests that the proportion of the pore space occupied by soil air is not available for flow (and could be visualised as being replaced by solid material) so that the unsaturated permeability would be less than the constant saturated permeability and would be a function of the moisture content. It can be argued (a) that the reduction of permeability with decreasing moisture content should be faster than linear since the larger pores empty first and $K$ should vary as the square of the effective diameter, and (b) that permeability should become negligible when soil water around the points of contact becomes isolated (Philip, 1969: 219).

Parametrization from the micro-scale of the soil particle to the macro scale of the Darcy continuum has been attempted in a number of ways and is not easy even for saturated flow. One approach is to assume a particle scale model of such simple geometry that the simplified Navier-Stokes equation can be solved directly. Scheidegger (1960) grouped such geometrical models into capillaric models, hydraulic radius models and drag models. A second approach of parameterizing from particle scale to pedon scale is the use of statistical models involving simple
assumptions in regard to the three factors involved: the nature of the particles in the ensemble, the dynamics of their interactions at the micro-scale, and the statistical distribution at the macro-scale. The third approach is based on the derivation of a logically consistent averaging procedure and its application to the macro-scale differential equation. All three approaches to the prediction of the saturated permeability lead to unspecified parameters at the macro-scale and must be supplemented by measurements at this scale. The added complications for unsaturated flow strongly suggest the desirability of some partial analysis of the problem. An obvious type of partial analysis to be tried is dimensional analysis.

2.3. FROM PARTIAL SCALE TO PEDON SCALE

The most notable use of dimensional analysis in the study of soil water movement was the work of Miller and Miller (1955, 1956). They derive macroscopic results from the assumption of exact microscopic similarity and its application to the matric potential-interface curvature relationship and to the simplified Navier-Stokes equation, both of which are linear.

Defining $\lambda$ as any specified characteristic length, the relationship of moisture content to matric potential for two (or more) geometrically similar media, with identical drying and wetting contact angles must be given by a common function $\theta_H$:

$$\theta = \theta_H \left( \frac{\lambda p}{\sigma} \right),$$

(2)

where $\theta_H$ is a hysteretic function. From a comparison of the Navier-Stokes equation and the Darcy equation it can be deduced that the conductivity $K(\theta)$ must be inversely proportional to the viscosity of the liquid and proportional to the square of the characteristic length $\lambda$. Thus for two similar media we have a second functional $K_H$:

$$\frac{\mu K}{\lambda^2} = K_H \left( \frac{\lambda p}{\sigma} \right)$$

(3)

as the link between the scaling of the matric potential – moisture content relationship and the hydraulic conductivity – moisture content relationship.

The bulk of the study of the unsaturated zone, whether by analysis or numerical computation or measurement of soil properties, has been done at the pedon scale in which the soil is assumed to be a continuum with prescribed relationships to moisture content of both matric potential and hydraulic conductivity. A soil pedon is a small volume of soil that extends downward to the lower limit of common rooting of the dominant native perennial plants, or the lower limit of the genetic horizons, whichever is the deeper (Soil Survey Staff, 1960). Analysis at this scale is essentially one-dimensional being concerned only with moisture fluxes in the vertical direction. Analysis at field scale seeks either (a) to parametrize two
pedon-scale parameters—hydraulic conductivity \((K)\) and either hydraulic diffusivity \((D)\) or differential moisture capacity \((C)\)—at the field scale or (b) to evaluate such effective field parameters as effective sorptivity \((S)\) and ultimate mean infiltration rate \(f_{\text{ult}}\).

2.3. PARTIAL ANALYSIS AT THE PEDON SCALE

The equation of continuity for vertical flow in a pedon is:

\[
\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} = 0, \tag{4}
\]

where \(\theta(z, t)\) is the local moisture content, \(q(z, t)\) is the face velocity taken vertically downward, \(z\) is the depth below the surface and \(t\) is the elapsed time. The equation of motion for vertical movement is

\[
q = -K(\theta) \left( \frac{\partial h}{\partial z} - 1 \right), \tag{5}
\]

where \(K(\theta)\) is the effective hydraulic conductivity and \(h(z, t)\) is the matric potential which is negative in the unsaturated zone. The above continuity and flow equations can be combined to give a single prognostic equation in moisture content \((\theta)\):

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ \frac{\partial h}{\partial z} - K(\theta) \right], \tag{6}
\]

which is often referred to as Richard's equation (Richards, 1931). If the matric potential \(h(\theta)\) and the hydraulic conductivity \(K(\theta)\) are both taken as single-valued functions of moisture content, then we can define hydraulic diffusivity \(D(\theta)\) as

\[
D(\theta) = K(\theta) \frac{\partial h}{\partial \theta} \tag{7}
\]

and combine Equations (6) and (7) in the single equation

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ D(\theta) \frac{\partial \theta}{\partial z} - K(\theta) \right], \tag{8}
\]

for the single unknown moisture content \(\theta(z, t)\).

Alternatively one can define the specific water capacity as

\[
C(\theta) = \frac{d\theta}{dh} \tag{9}
\]

and write Equation (6) as a single prognostic equation for soil moisture potential \(h(\theta)\):

\[
C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial z} \left[ K(h) \frac{\partial h}{\partial z} - K(h) \right] \tag{10}
\]

which has the same form as the Fokker-Planck equation used in certain heat conduction problems (Carslaw and Jaeger, 1946) and in the study of certain stochastic processes.
By using the standard methods of dimensional analysis (Kuhnel et al., 1990) the diffusivity form of Equation (8) can be written as

\[
\frac{\partial \theta'}{\partial t'} = \frac{D_0 T_0}{z_0^2} \frac{\partial}{\partial z'} \left[D'(\theta') \frac{\partial \theta'}{\partial z'}\right] - \frac{K_0 l_0}{\theta_{sat} z_0} \frac{\partial}{\partial z'} [K'(\theta')],
\]

which is completely dimensionless in respect of all variables and parameters. Initial and boundary conditions in terms of moisture content will be automatically normalised in terms of the saturated moisture content and boundary conditions in terms of fluxes can be rendered dimensionless by relating them to the reference hydraulic conductivity \(K_0\).

The final step is to choose values of the characteristic time \(t_0\) and the characteristic depth \(z_0\) in such a way as to facilitate obtaining as simple a solution as possible to the particular problem being studied. One choice of characteristic time and depth is to ensure the absorption of the parameters into the independent variables by writing

\[
t_0 = \frac{D_0}{K_0^2} \theta_{sat}^2.
\]

and

\[
z_0 = \frac{D_0}{K_0} \theta_{sat}.
\]

For this choice of scales Equation (11) becomes

\[
\frac{\partial \theta'}{\partial t'} = \frac{\partial}{\partial z'} \left[D'(\theta') \frac{\partial \theta'}{\partial z'}\right] - \frac{\partial}{\partial z'} [K'(\theta')]
\]

in which the scale parameters \(\theta_{sat}, D_0,\) and \(K_0\) have been absorbed into the independent variables, thus obtaining the canonical form of the equation and completing the first stage of the partial analysis. A similar partial analysis can be made of the water capacity form of the prognostic equation given by Equation (10).

2.4. SIMPLIFIED SOLUTIONS AT PEDON SCALE

The second stage of the partial analysis is to seek an insight into the nature of the complete solution by examining the nature of the solutions of simplified versions of Equation (13), or the equivalent canonical form of the water capacity equation.

If the assumption is made that \(K'\) is constant, then the last term of Equation (11) reduces to zero and a simple similarity solution is possible for any functional form of \(D'\). This solution can be expressed in the form

\[
z'(\theta', t') = \phi(\theta') t'^{1/2},
\]

where \(\phi(\theta')\) depends on the functional form of \(D'(\theta')\) and the initial soil moisture content \(\theta_0'\). Since the cumulative infiltration \(F(t)\) through the surface at any time minus the drainage at the bottom of the column must equal the increase in total
moisture content in the column, we can write

\[ F(t') - K_0 z' = \int_{0}^{t'} z' \, d\theta' = t'^{1/2} \int_{0}^{t'} \theta_0 \phi(\theta') \, d\theta' \]  

(15)

For a wide variety of functional forms \( D'(\theta) \), an explicit solution for \( \phi(\theta') \) can be found (Philip, 1960) and in many cases integrated to provide explicitly the second integral in Equation (15). In such cases the cumulative infiltration can be written as

\[ F(t) = St^{1/2} + f_{ult} t, \]  

(16)

where the sorptivity \( S \) depends on the functional form of \( D'(\theta') \) and the difference between the initial and final moisture contents at the surface. It can be shown that the variation in the sorptivity \( S \) for widely different assumptions about the diffusivity function \( D'(\theta') \) are relatively small (Philip, 1957; Dooge, 1973). Thus, while a wide difference between the assumptions on soil properties produces marked differences between soil moisture profiles, there may be only minor differences in the sorptivity parameter that controls the high rate of initial infiltration.

If an assumption of constant conductivity which leads to the above class of similarity solutions is replaced by the more realistic assumption of a linear variation of hydraulic conductivity with moisture content, an explicit solution for the standard boundary conditions can still be obtained but is of necessity more complex than Equation (16) (Philip, 1969). Explicit solutions are available for some other parameter variations (Sanders et al. 1986; Kühnel, 1989) but they involve complex transformations of the variables and result in weighty algebraic expressions. These explicit solutions apply to semi-infinite columns and constant initial moisture content which allows the amalgamation of the bottom boundary condition and the initial condition. For a finite depth of profile, each term in these explicit solutions must be generalized to an infinite series. It would appear sufficient to use one of the simplified solutions in a partial analysis of the effects of spatial variability on infiltration and evaporation.

Even if we restrict ourselves to constant or linear conductivity, the serious problem still remains of the functional description of the initial soil moisture profile. For either wetting or drying over a prolonged period, the problem can be handled without serious error assuming constant initial moisture content in the whole profile but switching from wetting to drying and vice versa still present a major difficulty. If instead of assuming instantaneous ponding following the onset of rain, the pre-ponding phase (when infiltration equals rainfall) is taken into account, then the assumption in the post ponding solution of a constant initial moisture content is no longer valid. This can be overcome by matching the volumes of cumulative infiltration at the instant of ponding for the two cases. For any given model the profiles are very similar and consequently the minor redistribution involved will not cause serious error (Kühnel, 1989).
3. Partial Analysis at Field Scale

3.1. Limiting Forms of Analytical Solutions

To tackle the problem of spatial variability without producing an impenetrable jungle of complex algebra requires the assumption of both a simplified model of the parameter variation in the pedon scale parameters and a simplified model of the soil moisture behaviour at pedon scale. The soil similarity hypothesis of Miller and Miller (1956) provides a reasonable basis for reducing the spatial variability of the various soil parameters and relationships to the single spatial variability of a scale factor. The simplification of the predicted moisture fluxes at pedon scale can be attempted by seeking simplified solutions that are reasonably realistic and in particular by seeking limiting forms that bound the space occupied by more complex solutions.

The available closed form solutions of the Richards equation for both constant flux and constant concentration boundary conditions have been reviewed by Kühnel (1989) for both infiltration and evaporation. These can be grouped according to the diffusivity function as (a) delta function $D$ solutions; (b) constant $D$ solutions; and (c) Fujita $D$ solutions. Of these three groups, only the first two would be sufficiently simple for the purpose of partial analysis. Within the second group, the solution will vary according as the unsaturated hydraulic conductivity is taken as constant or linear or quadratic. Again only the first two cases are simple enough for partial analysis. The candidate closed form solutions for partial analysis thus reduce to three models: (1) delta function $D$ and arbitrary $K$; (2) constant $D$ and constant $K$; (3) constant $D$ and linear $K$. Since diffusivity is defined as the product of hydraulic conductivity and the rate of variation of matric potential with moisture content, each of these simplified models implies a specific form of the characteristic curve relating matric potential and soil moisture retained. Thus the model of constant $D$ and linear $K$ necessarily implies an exponential variation of moisture content with matric potential which was the form in which the model was originally proposed by Gardner (1958). This model has been widely used by soil physicists.

A comparison of simplified solutions of the horizontal absorption problem and the ponded infiltration problem was made by Philip (1969 pp. 270–275). For the absorption problem, Philip argued that since $D(\theta)$ in real media would have a shape intermediate between those for a matched delta function and a matched effective constant value of $D$, the delta function solution and the constant $D$ solution would “form an envelope of possible solutions”. For the infiltration case where the gravity term must be allowed for, he concluded:

“For infiltration phenomena with both linearised and delta-function solutions available, either gives an estimate of the integral behaviour of any real medium. An estimate of the order of magnitude of the error is provided by the difference between the two solutions”
This example of partial analysis is relevant to our subject since surface fluxes are part of the integral behaviour of a soil column. A comparison of the delta function solution and the constant $D$ – constant $K$ solution of the ponded infiltration problem was made by Dooge (1973: pp. 278–281). The two separate comparisons can be combined and compared with the four term non-linear solution for Yolo light clay calculated by Philip (1969: pp. 248–255). The results indicate that the non-linear series solution is quite closely bounded by the delta-function solution and the constant $D$ – constant $K$ solution (Kuhnel et al., 1990).

3.2. Spatial variability in pedon parameters

Until recently, most of the papers published on the application of soil physics to infiltration and evaporation were concerned with the behaviour at pedon scale, i.e. with conditions of vertical flow at a continuum point in a horizontal plane. Though variability of soil properties was known to be considerable (Beckett and Webster, 1971), the problem of allowing for such variations has only been tackled in the past fifteen years. The simplification arising from Miller and Miller scaling has made this feasible.

Warrick et al. (1978) applied the Miller and Miller concept of scaling to data of Nielsen et al. (1973) at 20 locations in a 150 ha field. The samples at different soil depths were taken as different locations thus giving a set of 120 sample points in all. The data for matric potential at each sample point (840 data points in all) were then analysed to determine the best set of scaling factors and polynomial coefficients to represent the 840 data points by a single matric-potential soil moisture relationship. The sum of the squares of the deviations was reduced by such scaling to 15% of the value for the unscaled data. The data for the hydraulic conductivity was similarly processed. Warrick et al. (1977) applied a similar procedure to the data from 36 sites in an 87 ha field and 8 sites within 7 km of one another. It was found that the values of the scaling factor derived from data on matric potential were more reliable than those derived from hydraulic conductivity. The distribution of scaling factors were found to be approximately log normal. Further studies of the variation in pedon scale parameters were made by Simmons et al. (1978) and by Reichardt and colleagues (1973 and 1975).

3.3. Spatial variability in field parameters

At field scale we are concerned not so much with the pedon scale parameters of matric potential, hydraulic conductivity and hydraulic diffusivity but with such field scale parameters as sorptivity and ultimate rate of infiltration, with time to surface ponding after rain starts and time to surface drying after it ends, and with the values of parameters to be used in conceptual models of hydrologic processes. All three simplified models discussed in the last section give the cumulative volume of infiltration immediately following instantaneous ponding as

$$F(t) = St^{1/2} + f_{ult}t,$$  \hfill (17)

where $S$ is the sorptivity (Philip, 1969) and $f_{ult}$ is the final rate of surface infiltration.
For a soil characterised by parameters $K_{sat}$ and $D_0$ and dimensionless relationships $h(t)$ and $K(\theta)$, dimensional analysis would suggest that Equation (17) can be written as

$$\frac{K_{sat}}{D_0} F(t) = \frac{S}{\sqrt{D_0}} \left( \frac{K_s^2 t}{D_0} \right)^{1/2} + \frac{f_{ult}}{K_{sat}} \left( \frac{K_s^2 t}{D_0} \right). \quad (18)$$

This formulation indicates that the spatial variability of the sorptivity $S$ should correspond to the square root of the variability of the diffusivity and the variability of the ultimate infiltration rate should correspond to the variability of the hydraulic conductivity.

Some authors have applied scale analysis directly to the sorptivity and the ultimate rate of infiltration at the field scale. The advantage in dealing directly with the initial high rate infiltration is offset by the fact that the sorptivity is dependent on the initial moisture content. Sharma et al. (1980) studied the variation in a 9.6 ha sub-catchment in Oklahoma grassland where infiltration was measured at 26 locations. Though there was no significant difference between the results for the three types of silt loam present, there were wide differences in water retention, hydraulic conductivity and infiltration data. Vauclin et al. (1981) investigated the variation in physical properties at 17 points in a one hectare field of bare soil and reduced the variability of $S$ and $f_{ult}$ by Miller and Miller scaling as indicated by Figure 4. Youngs and Price (1981) extended scaling analysis to the case of dissimilar soils.

### 3.4. LIMITING VALUES OF TIME TO PONDING

The time to ponding for constant rainfall can be readily expressed in closed form for the models mentioned in Section 3.1. For any given model, the time to ponding is naturally a function of the ratio of the rate of rainfall to the saturated conductivity. The dimensionless time to ponding varies little for the more complex three-parameter models. For the simpler models the differences are greater because the shapes of the developing soil moisture profile differ widely from actual moisture profiles. If both atmosphere-controlled infiltration prior to ponding and soil-controlled infiltration following ponding are to be simulated by the same simple model, then it may be necessary to tune one parameter of the model (e.g. effective conductivity in the constant $D$ - constant $K$ model) to get a good representation of both types of infiltration.

A study of the switching between atmosphere-control and soil-control at a field scale would involve a study of the distribution of the time to ponding and the time to surface dryness over the field. One approach might be to assume a simple model for soil moisture flux in a vertical direction and combine this with a simple assumption for the spatial distribution of the scale factor for similarity of soil geometry. Such a procedure assumes through its use of a one-dimensional vertical analysis that all of the individual pedons and their underlying profiles are independent of one another. In practice there would be lateral redistribution of soil moisture wherever a lateral gradient of matric potential arose.
Fig. 4. Field scale solutions for ponded infiltration (Vauclin et al., 1981).
Partial analysis would treat the approach described in the last paragraph as a limiting solution corresponding to the assumption of zero horizontal hydraulic conductivity. The strategy of partial analysis would be to seek another limiting solution which would complete a pair of limiting solutions capable of bounding the family of more realistic solutions. The assumption of infinite horizontal hydraulic conductivity suggests itself as a candidate for such a role. Such an assumption implies that in a heterogeneous field the matric potential (but not the moisture content) at any given level would be a constant thus facilitating an analytical solution. The fact that the matric potential is constant at the surface at all times means that all points of the surface must reach saturation simultaneously so that there is no variation in the time to ponding throughout such a heterogeneous field. For realistic cases, the distribution of time to ponding should vary between these two limiting cases.

4. Basin Scale and Biome Scale

4.1. Spatial Variability and Catchment Water Balance

Classical models of surface runoff generation were based on the infiltration concept due to Horton (1933). In these early models the whole catchment was considered as a unit and thus the predicted surface runoff was everywhere zero in the pre-ponding phase when rainfall was less than the high rate of initial infiltration capacity. Similarly evaporation was considered to be everywhere the same being at the potential rate as long as soil moisture exceeded the “field capacity” and varying according to some empirical rule with the soil moisture values when these were less than the field capacity. Though many assumptions were made about the relationship between actual evaporation and field moisture content, the most popular of these in computer models of climate has been the linear variation proposed independently by Thornthwaite and Mather (1955) and by Budyko (1955).

Penman in 1951 introduced the concept of spatial variability by dividing the catchment into riparian areas and non-riparian areas. He suggested that in the riparian areas the water table would be close enough to the surface for evaporation to be always at the potential rate and for direct recharge of the groundwater by rainfall; whereas for the non-riparian areas the groundwater would be well below the surface and the state of the unsaturated zone would affect both the actual rate of evapotranspiration during dry periods and the actual rate of infiltration during wet periods. This simple division of the area represents a good example of a partial analysis of the complex problem of the effect of spatial variability on land surface fluxes. This approach was extended by Hewlett (1961) who suggested that for hillslopes most of the rainfall would infiltrate through the surface but that a high proportion of this infiltrated water would flow laterally and either produce saturation (and consequently overland surface runoff) at a lower elevation or else flow laterally into the drainage network. This alternative model to the Hortonian model of local overland flow implies that the extent of
riparian areas adjacent to the drainage network is not fixed but increases during the storm as the soil becomes saturated in areas of shallow groundwater and decreases during the recession of the resulting runoff event. Reviews of the development of this approach have been given by Dunne (1982), Kirkby (1985) and by Troendle (1985).

Many variations of catchment modelling have been tried between the two extremes of a simple division of the catchment area into two sub-areas each with its own set of parameters and a continuous variation of parameters usually based on some statistical distribution of the geometrical scale factor. Thus Gleick (1987) in a climate study on the simulation of monthly flows of the Sacramento River in California substantially improved the performance of the model of catchment response by dividing the catchment into two sub-catchments based largely on topographic factors. Dunin and Astin (1981) by dividing the catchment into 3 domains improved the predictive capacity of the WATSIM catchment response model on an experimental catchment of 5 ha in New South Wales from 54 to 81% for weekly flows in a dry year and from 88 to 96% for weekly flows in a wet year. They found that precision for differentiating between domains was lost if a large number of domains was used.

Peck et al. (1977) using a normal distribution with a coefficient of variation of 0.25 for a 97-ha catchment in Eastern Tennessee, found no appreciable difference in the water balance elements compared with the use of mean values. In contrast, Sharma and Luxmoore (1979) using Monte Carlo simulation found that the amount of runoff was sensitive to the value of the scaling factor and linked this with the variable source area concept of runoff generation (Dunne et al., 1975). They also found in contrast to Peck et al. (1977), that the water balance simulated by the mean scale factor was significantly different from the integrated response for both the log-normal and the normal distribution. Freeze (1980) in another Monte Carlo simulation of spatial variability found that the statistical properties of the generated runoff would be greatly in error if the mean hydraulic conductivity were used.

4.2. Conceptual Models of Catchment Response

The hydrological behaviour at the catchment scale is usually studied on the basis of conceptual models of catchment response in which the various processes are simulated by a simple arrangement of lumped components each of which is a simplified representation of a hydrological process. The lumped parameters of these models are optimised on the basis of field data but this can be an unreliable process (Dawdy and O'Donnell, 1965). There are many such conceptual models (Fleming, 1975) and the more complex of them do not always outperform the simpler ones (WMO, 1975). The soil-moisture accounting component and the linked algorithms for computing the surface fluxes of actual infiltration and actual evaporation are usually the weakest features of such conceptual models.

It was pointed out over twenty five years ago by Kohler (1963) that there is an inherent conflict in the conceptual modelling of soil moisture between a desire to improve vertical resolution by a multi-layer model and a desire to improve horizontal
resolution through multi-capacity soil-moisture accounting. In the earlier conceptual models of catchment response to spatial variation in infiltration capacity was taken as a uniform distribution from zero infiltration capacity (i.e. an impermeable surface) to a prescribed maximum infiltration capacity. The latter parameter was fixed either using some field data on soil properties or by optimisation using rainfall and runoff data or in some other way. A linear cumulative distribution corresponding to this assumption is used in the well-known Stanford Model (Crawford and Linsley, 1966). Most of the recent models use a power-law distribution which gave improved results in applications by Popov (1962) in the Soviet Union, by the East China College of Hydraulic Engineering (ECCHE, 1977) in the humid areas of South China, and by Clark (1980) in Australia.

The spatial variability in the process of surface-runoff has been explicitly addressed in some of the commonly used conceptual models of catchment response. Kibler and Woolhiser (1970) used a deterministic approach based on the kinematic wave approximation to study the effects of the variation of slope between the elements of a cascade of plane surfaces. Machado and O'Donnell (1982) used stochastic modelling to study the effect on runoff from plane surface when the relevant parameters (length, slope, roughness, infiltration parameters) were selected at random from a normal distribution. Beven and Kirkby (1979) developed an exponential store model (TOPMODEL) which would take care of changes in topography expressed in terms of the ratio of area to slope rather than simple area as in classical conceptual models. This approach can be extended to include also transmissivity thus producing a spatially variable topography-soils index (Famigliatti and Wood, 1990). TOPMODEL was developed to allow for spatial variability and was tested on some small catchments in Britain (Beven et al., 1984). The problem of scale in the runoff process was the main topic of two previous workshops in January 1982 at Caracas (Rodriguez-Iturbe and Gupta, 1983) and in November 1984 at Princeton (Gupta et al., 1986). While the main topic running through the published papers was quantitative geomorphology, a number of them are relevant to the question of spatial variability.

4.3. FROM CATCHMENT SCALE TO GCM SCALE

As we move from the catchment scale to the regional scale and the scale of general circulation models (over 100 km), our concepts and results become less assured. What is clear is that there is scope both for improving the interaction between hydrologic models and atmospheric models and also for improving the performance of both types of models by carefully planned experiments both on the computer and in the field. One candidate for such an experiment is the advection-aridity approach described below.

The difficulties of allowing for the high spatial variability in soil and vegetation in the estimation of evapotranspiration has led to the proposal of an approach for the estimation of regional evapotranspiration which depends on the fact that the atmospheric variables near the land surface will be more uniform over a wide area
than the local soil and vegetation properties (Bouchet, 1963; Morton, 1965; Brutsaert and Stricker, 1979). In this advection-aridity approach a distinction is made between the potential evaporation under potential conditions \((E_{po})\) and the estimated potential evaporation under non-potential condition \((E_p)\). In the former case, water is not limiting and consequently the actual evaporation \((E)\) is equal to the potential evaporation under potential conditions \((E_{po})\) i.e.:

\[
E = E_{po}.
\] (19)

Under non-potential conditions, less energy is used in the form of the latent heat of vaporisation \((LE)\) and the amount of energy thus made available \((q)\):

\[
q = L(E_{po} - E)
\] (20)
can be used for the transfer of sensible heat or for the creation of turbulence etc., near the surface of the ground. Such changes will increase the apparent potential evaporation \((W_p)\) based on a combination type formulae which has been calibrated for potential evaporating conditions. Consequently, if the general circulation of the atmosphere outside the planetary boundary layer remains unaffected by the changed conditions, the apparent potential evaporation \((E_p)\) increases by an amount corresponding the extra energy made available, i.e.:

\[
E_p = E_{po} + \frac{q}{L}.
\] (21)

Combining the relationship given by Equations (20) and (21) we obtain the final relationship for the actual evaporation under non-potential conditions:

\[
E = 2E_{po} - E_p
\] (22)

which indicates that the actual evaporation \((E)\) decreases as the apparent potential evapotranspiration \((E_p)\) increases. For the application of this method, it is necessary to estimate both the potential evaporation \((E_{po})\) from climatological data and the potential evaporation under potential conditions \((E_{po})\) for the area concerned (Brutsaert and Stricker, 1979).

This simplified Bouchet–Morton approach to the estimation of regional evaporation can be considered as a partial analysis of the problem of interaction between the unsaturated subsurface zone and the planetary boundary layer. Well planned experiments on general circulation models could help to evaluate the advective-aridity hypothesis of Bouchet and Morton on the basis of the behaviour of the lowest layer of these multi-layer models.

4.4. ACTUAL EVAPORATION AT THE BIOME SCALE

The problem of estimating actual evaporation at the GCM scale only arises for the case of soil moisture depletion. In very humid areas, the evaporation will be close to the potential rate which can be computed internally by the general circulation model. On the other hand, for very arid conditions, there would be no surface runoff and no recharge to groundwater and as a result the actual evaporation
would be equal to the precipitation which is also computed internally by the atmospheric model.

If the hypothesis is made that the vegetation at equilibrium depends only on the rainfall and the net radiation at the surface, then dimensional analysis suggests that the ratio of actual to potential evaporation will be a function of the long term ratio of rainfall to potential evaporation. A number of empirical relationships have been proposed which express this relationship in a functional form that satisfies the two limiting conditions described above and approximates the long term water balance of the catchment examined. (Schreiber, 1904; Ol’dekop, 1911; Budyko, 1948, 1961; Turc, 1954; Pike, 1964). Figure 5 shows the exponential relationship (a) suggested by Schreiber (1904) and hyperbolic tangent relationship (b) suggested by Ol’dekop (1911). Budyko (1961) found the data for over a thousand catchments in the USSR fell between these two limits and suggested the use of their geometric mean as estimating the actual evaporation within ±10%. The simpler relationship of Turc (1954) as slightly modified by Pike (1964) approximates this geometric mean very closely. The Turc–Pike relationship is:

\[ AE = \frac{P}{1 + \left(\frac{P}{PE}\right)^2}^{1/2} \]  

where \( AE \) is the long term actual evaporation, \( P \) is the long term precipitation and \( PE \) is the long term potential evaporation.

A remarkable attempt has been made by Eagleson and his co-workers to establish hydrologic laws on the macro-scale. Eagleson (1978) used a stochastic-dynamic approach to this problem of the long-term water balance. He assumed representative probability density functions for such climatic variables as the interval between storms, the duration of storms, the intensity of rainfall in the storms and the potential evaporation. Combining these assumed probability density functions with simplified assumptions in regard to the hydrological processes involved in the conversion of storm rainfall to runoff, Eagleson (1978) derived the probability density function of the actual infiltration during storms and the probability density function of the actual evaporation between storms. The average volume of infiltration and the average volume of evaporation can then be found from the average number of storms and used to construct the long-term water balance. Using this approach, Eagleson (1978) was able to relate the key ratio of actual to potential evaporation to five parameters; three of these parameters related to the properties of soil and two of them to the properties of the vegetation. The resulting relationship has the same general form as the Turc–Pike relationship shown on Figure 5 but has the additional advantage that the effect of changes in the soil parameters and the vegetation parameters and in the statistical assumptions about climatic conditions can be systematically studied in order to gain insight into the sensitivity of the equilibrium to these parameters.

In a further development of this approach to the long-term water balance, Eagleson (1982b) suggested the existence of equilibrium relationships between the
Eagleson introduced two hypotheses, one relating to the situation where the vegetation is water-limited and the second where the vegetation is energy-limited. Eagleson (1982b) suggested that under conditions of water-limitation, a system of vegetation would for the given climate and soil moisture conditions produce the particular canopy density which reduced moisture stress at the roots to a minimum. For the case where vegetation activities are limited by energy rather than by water, Eagleson (1982b) suggested that the vegetative system would tend to maximize the biomass for the given amount of energy. By applying these two hypotheses to his 1978 stochastic-dynamic model, Eagleson (1982b) derived the equilibrium
relationship defining the limiting curves relating the ratio of actual to potential evaporation (which is species-dependent) to the density of the vegetative canopy. Preliminary comparison of data for a few catchments in humid and semi-arid regions tends to confirm the derived limiting relationships as reasonable (Eagleson and Tellers, 1982). In a further development (Eagleson and Segarra, 1986), these hypotheses of ecological optimality were applied to the effect of climate change on the annual water balance of Savanna vegetation and again the results are encouraging.

The use of general circulation models to study the feedback of vegetation on climate has been reviewed by Rowntree (1984). In order to improve our understanding of climate change and vegetation, Rowntree recommended further observational studies, improvements in GCM parametrizations, and GCM experiments, particularly in regard to albedo, hydrology and surface roughness. He recommended (1) the observation and modelling of the dependence of albedos on soil moisture as well as on vegetation, (2) the parametrization of hydrological processes such as runoff, evaporation of intercepted rainfall, root extraction, etc., at the GCM scale, taking into account the inhomogeneities of soils, slopes, vegetation, precipitation, etc., at the microscale, and (3) the observation and modelling of the surface roughness and surface stress of vegetation.

Meanwhile, some climate modellers have been seeking to improve their representation of hydrologic processes and the resulting surface fluxes. Reviews of practice in the past have been given by Carson (1982) and by Dickinson (1984b). To avoid the complexity of simulation based on realistic bio-physics, Dickinson (1984a) and Sellars (1986, 1987) have proposed simple biosphere models consisting of a single canopy and a single trunk.

The effect of spatial variability of soil properties and of vegetation on the water balance was studied by Milly and Eagleson (1982). In a further study, Entekhabi and Eagleson (1989) examined the parametrization of spatial variable hydrologic processes in general circulation models in order to explore the major sensitivity of these models to soil type and climatic forcing.

4.5. SENSITIVITY OF CATCHMENT RUNOFF TO CLIMATE CHANGE

The effect of climate change due to the increase in atmospheric CO$_2$ or other factors on the water balance of catchments in general and on runoff in particular to estimate (National Research Council, 1977; Klemes, 1985; Dooge, 1986). Since runoff is the difference between precipitation and actual evaporation there is likely to be a proportionally larger change in runoff if any change occurs in either precipitation or potential evaporation. We can define the positive magnification $R_p$ due to a change in precipitation as

$$\frac{\Delta Q}{Q} = R_p \frac{\Delta P}{P}$$ (24)
and the negative magnification $R_{PE}$ due to a change in potential evaporation as

$$\frac{\Delta Q}{Q} = R_{PE} \frac{\Delta PE}{PE}. \quad (25)$$

It can be shown that if the equilibrium condition is such that

$$\frac{AE}{PE} = \Phi \left( \frac{P}{PE} \right) \quad (26)$$

as shown on Figure 5, then these two magnification factors must satisfy the condition

$$R_P + R_{PE} = 1. \quad (27)$$

For very humid conditions $R_P = 1$ for all models shown on Figure 5 but for very arid condition $R_P$ approaches infinity for the Schreiber model compared with $R_P = 3$ for the other two models.

A crucial point in predicting the effect of CO$_2$ doubling on streamflow is whether the direct effect of CO$_2$ in reducing evaporation at a plot scale would also apply at catchment scale (Strain and Cure, 1985; Shugart et al., 1986). Thus, Idso and Brazel (1984) compared the effect of a 2% increase in temperature and 10% decrease of precipitation on the runoff from Arizona streams for the two cases of no direct CO$_2$ effect and full effect of CO$_2$ on transpiration at catchment scale. Without any direct CO$_2$ effect the flow was estimated to decrease by 41%; with the full effect the flow was estimated to increase by 42%. This is only one of many problems still to be solved by interdisciplinary cooperation between climate modellers and hydrologists and biologists. To solve this and other problems will require a notable co-operative effort involving analysis, numerical experimentation, field observation, and above all meaningful communication.

References


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