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Characterization of leaky faults: Study of water flow in aquifer-fault-aquifer systems

Chao Shan, Iraj Javandel, and Paul A. Witherspoon

Earth Sciences Division, Lawrence Berkeley Laboratory, University of California, Berkeley

Abstract. Leaky faults provide important flow paths for fluids to move underground. It is often necessary to characterize such faults in engineering projects such as deep well injection of waste liquids, underground natural gas storage, and radioactive waste isolation. To provide this characterization, analytical solutions are presented for groundwater flow through saturated aquifer-fault-aquifer systems assuming that both the aquifers and the fault are homogeneous and that the fault has an insignificant effect on aquifer hydraulic properties. Three different conditions are considered: (1) drawdown in the unpumped aquifer is negligibly small; (2) drawdown in the unpumped aquifer is significant, and the two aquifers have the same diffusivity; and (3) drawdown in the unpumped aquifer is significant, and the two aquifers have different diffusivities. Methods are presented to determine the fault transmissivity from pumping test data.

Introduction

Faults are fractures along which significant movement has occurred. During this movement, the rock adjacent to the fault is often pulverized or ground into a clayey, soft material called "fault gouge." In some instances the material in the fault zone may be broken and sheared, creating a "fault zone breccia." Fault gouge usually has low permeability. Alternatively, the fault zone breccia may be highly permeable [*Birkeland and Larson*, 1978]. In groundwater hydrology and petroleum engineering, faults are classified according to their hydraulic response during a field pumping test: (1) tight faults ($K \approx 0$), (2) constant head faults ($K \rightarrow \infty$), or (3) nonconstant head leaky faults (or, simply, "leaky faults," for which $0 < K < \infty$), where K is the hydraulic conductivity of the fault [*Witherspoon et al.*, 1967]. Tight faults are those that cut through an aquifer and, for all practical purposes, hydraulically separate one part of the aquifer from the rest of the system and from other water-bearing formations above and below. Constant head faults are those that connect the aquifer to a large constant head body of water; the path along the fault between the aquifer and the water body has practically an infinite permeability. In reality, many faults are leaky and have finite hydraulic conductivity. For many engineering projects that are concerned with leaky faults, one of the most important questions is, How leaky is the fault?

For many years, deep well injection has been considered a cheap and safe way of disposing of waste liquids by industry. In 1983 an estimated 423 million gal. (1.6 million m^3) of waste water were injected into 181 wells in the United States [*U.S. Environmental Protection Agency*, 1985]. Storing natural gas in aquifers is an effective way to accommodate the significant seasonal difference in natural gas consumption. More recently, constructing huge repositories in a low-permeability unsaturated zone in an arid region is thought to be a practical option for disposing of high-level radioactive waste. In all these projects, if there is a leaky fault nearby, there is the potential for excursion of stored or contaminated fluids.

The importance of leaky fault effects on regional groundwater flow has been addressed by several authors in previous investigations. *Ganser* [1987] discussed the hydrogeologic characteristics of the Ramapo Fault in northern New Jersey. In his work, "geologic mapping and geophysical techniques were employed to locate the fault prior to core drilling and packer testing of the fault" (p. 664), and the calculated hydraulic conductivity of the Ramapo Fault was less than 10^{-7} cm s^{-1}. Another example is an investigation by *Snipes et al.* [1986] of the hydrogeology of the Pax Mountain Fault zone in South Carolina. One of the main discoveries they report is the significant heterogeneity of the fault zone. In fact, most fault zones are highly heterogeneous [*Birkeland and Larson*, 1978]. Because of this, a laboratory test on core samples may not be sufficient to determine the hydraulic properties of a leaky fault. Thus, to characterize a leaky fault, one needs to rely on field tests, and to design an effective field test, it is necessary to establish a relationship between the physical properties of the system and the hydraulic response associated with the fault.

Using the method of images, *Ferris* [1949] and *Jacob* [1950] proposed a method for calculating drawdowns in a pumped aquifer that is intersected by tight or constant head faults. Ferris also presented a method to locate an impervious boundary, such as a tight fault, by using drawdown data at two observation wells. The effect of a leaky fault was studied by *Yaxley* [1987]. Based on a previous study of an aquifer with a linear discontinuity [*Bixel et al.*, 1963], Yaxley developed a mathematical model that describes the effect of a partially communicating fault on transient pressure behavior. In his model the two parts of the system, the aquifer and the fault, have finite but different hydrologic properties. Analytical solutions were obtained based on the assumption of a linear pressure distribution across the fault zone. His result can be used to determine the horizontal transmissivity of the fault section inside the aquifer. However, a fault zone is usually anisotropic, with its vertical permeability greater than its horizontal permeability [*Birkeland and Larson*, 1978]. Therefore it is still necessary to obtain new solutions for characterizing leaky faults.

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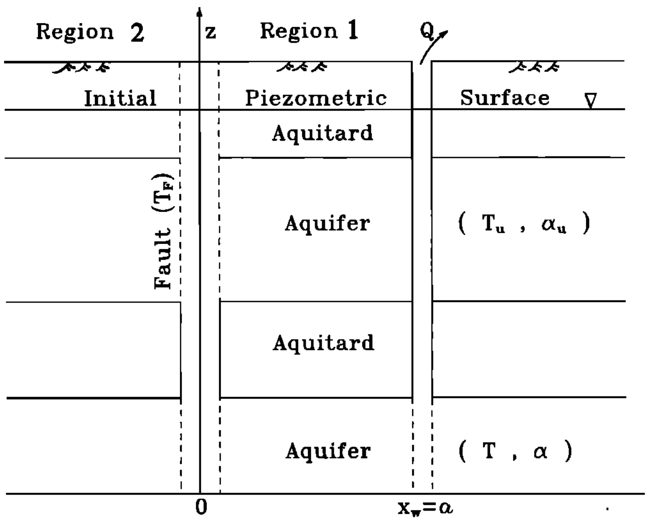

Figure 1. Schematic view of the aquifer-fault-aquifer system.

Here we develop analytical solutions for saturated flow in an aquifer-fault-aquifer system. When water is pumped from one of the two aquifers, there are two possible responses for the unpumped aquifer depending on the ratios of transmissivity between fault and aquifers: (1) drawdown in the unpumped aquifer is insignificant or too small to measure, or (2) drawdown in the unpumped aquifer is measurable and significant. For both cases we will first derive the analytical solutions for drawdowns in the aquifers and then discuss methods for determining the transmissivity of the leaky fault.

Transient Solutions

Consider a system of two horizontally extensive aquifers separated by an aquitard and intersected by a leaky fault with a finite hydraulic transmissivity, T_F (Figure 1). We assume both aquifers are homogeneous, isotropic, and of uniform thickness. A well fully penetrates one of the aquifers and is pumped at a constant rate, Q. In Figure 1 we have assumed that the well is screened in the lower aquifer. However, the solutions obtained in this study are also applicable to the case where the well is screened only in the upper aquifer. For generality, we will use the terms "pumped aquifer" and "unpumped aquifer" in the following to distinguish the two aquifers. For a near vertical fault as shown in Figure 1, we choose Cartesian coordinates such that the x-y plane is in the pumped aquifer with the origin at the fault, the x axis passes through the pumping well, and the z axis is along the fault, positive towards the unpumped aquifer. The fault divides both aquifers into two different regions. That part of the aquifer on the pumping side is designated as region 1, and that on the other side, as region 2. To simplify the problem, we also assume that the system is initially at equilibrium.

There are two possible responses for the unpumped aquifer: (1) drawdown in the unpumped aquifer is negligibly small such that a constant head (or zero drawdown) can be assumed for this aquifer, and (2) drawdown in the unpumped aquifer is significant. Case 1 may be found where the unpumped aquifer has a much larger transmissivity than the pumped aquifer, or the unpumped aquifer is overlaid by a large water body such as a river or a lake near the fault. Under these circumstances, we need to obtain an analytical solution for drawdown at an arbitrary point (x, y) in the pumped aquifer only. For case 2, however, we need to solve for drawdowns in both aquifers.

Case 1: Zero Drawdown in Unpumped Aquifer

Mathematically, the problem can be formulated by the following governing equations, with initial and boundary conditions as shown:

$$\frac{\partial s_1}{\partial t} = \alpha\left(\frac{\partial^2 s_1}{\partial x^2} + \frac{\partial^2 s_1}{\partial y^2}\right) + \frac{\alpha Q}{T}\,\delta(x-a)\delta(y) \tag{1}$$

$$\frac{\partial s_2}{\partial t} = \alpha\left(\frac{\partial^2 s_2}{\partial x^2} + \frac{\partial^2 s_2}{\partial y^2}\right) \tag{2}$$

$$s_1(x, y, 0) = s_2(x, y, 0) = 0 \tag{3}$$

$$s_1(x, \pm\infty, t) = s_2(x, \pm\infty, t) = 0 \tag{4}$$

$$s_1(+\infty, y, t) = s_2(-\infty, y, t) = 0 \tag{5}$$

$$s_1(0, y, t) = s_2(0, y, t) \tag{6}$$

$$T\frac{\partial s_1}{\partial x} = T\frac{\partial s_2}{\partial x} + T_F\frac{s_2}{L} \qquad x = 0 \tag{7}$$

where s_1 and s_2 represent the drawdowns at any arbitrary points in regions 1 and 2, respectively; t is time; L is the length of the flow path in the vertical direction along the fault; $\delta(x - a)\delta(y)$ is the Dirac δ function; a is the distance from the pumping well to the fault; and α and T are diffusivity and transmissivity, respectively, defined as

$$\alpha = K/S_s \qquad T = KH \tag{8}$$

where S_s is the specific storage of the pumped aquifer, K represents hydraulic conductivity, and H represents thickness. The subscript F distinguishes the transmissivity of the fault from that of the pumped aquifer.

The second term in (7) represents the flow component along the fault in the z direction, in which we have neglected the storage of the fault. The validity of this approximation has been demonstrated by comparing the solution with an exact solution for the case where the transmissivity of the fault is equal to that of the pumped aquifer [*Shan*, 1990].

For the governing equations, by first taking the Laplace transforms of (1) and (2) with respect to t, and then taking the exponential Fourier transform with respect to y, we obtain

$$\frac{d^2 w_1}{dx^2} - A^2 w_1 = -\frac{Q\cdot\delta(x-a)}{pT} \tag{9}$$

$$\frac{d^2 w_2}{dx^2} - A^2 w_2 = 0 \tag{10}$$

where

$$w(x, \rho, p) = \mathcal{F}\{v(x, y, p)\} = \int_{-\infty}^{+\infty} v(x, y, p)\exp(i\rho y)\,dy \tag{11}$$

$$v(x, y, p) = \mathcal{L}\{s(x, y, t)\} = \int_0^{\infty} s(x, y, t)\exp(-pt)\,dt \tag{12}$$

$$A = (p/\alpha + \rho^2)^{1/2} \tag{13}$$

where p and ρ are the variables of the Laplace and the exponential Fourier transforms, respectively. In the above process we have used (3) and (4). The remaining boundary conditions (5), (6), and (7) in the transformed domain are

$$w_1(+\infty, \rho, p) = w_2(-\infty, \rho, p) = 0 \tag{14}$$

$$w_1(0, \rho, p) = w_2(0, \rho, p) \tag{15}$$

$$T\frac{dw_1}{dx} = T\frac{dw_2}{dx} + T_F\frac{w_2}{L} \qquad x = 0 \tag{16}$$

The solutions to (9) and (10) that satisfy (14) through (16) are

$$w_1 = \frac{Q}{2T}\left[\frac{\exp(-|x-a|A)}{pA} - \frac{c\exp[-(x+a)A]}{pA(A+c)}\right] \tag{17}$$

$$w_2 = \frac{Q}{2T}\cdot\frac{\exp[-(a-x)A]}{p(A+c)} \tag{18}$$

where c is a parameter defined by

$$c = \frac{1}{2L}\cdot\frac{T_F}{T} \tag{19}$$

The inversions of (17) and (18) give the solutions for drawdowns in the two regions, which can be written in the following unified form:

$$s = \frac{Q}{4\pi T}W\left[\frac{(x-a)^2+y^2}{4\alpha t}\right] - \frac{Qc\alpha^{1/2}\exp[c(|x|+a)]}{4\sqrt{\pi}T}\int_0^t g(\tau)\,d\tau \tag{20}$$

where the integrand is defined as

$$g(\tau) = \frac{e^{c^2\alpha\tau - y^2/(4\alpha\tau)}}{\tau^{1/2}}\operatorname{erfc}\left(\frac{|x|+a}{2(\alpha\tau)^{1/2}} + c\sqrt{\alpha\tau}\right) \tag{21}$$

In (20), s represents both s_1 and s_2; $W(u)$ is the well function defined by

$$W(u) = \int_u^\infty \frac{e^{-x}}{x}\,dx \tag{22}$$

The first term in (20) is obviously the *Theis* [1935] solution, while the second term is an adjustment for the effects of leakage through the fault. If $T_F = 0$, there is no hydraulic connection between the two aquifers, and the solution is reduced to the Theis solution. Details of the derivation process leading to (20) are given in the appendix.

By using (20), one can calculate drawdowns in the pumped aquifer. An example is given in Figure 2, which shows three depletion curves at $t = 10$ hours along the $x - z$ plane. The parameters used in this example are $a = 100$ m, $Q = 0.005$ m^3 s^{-1}, $K = 10^{-4}$ m s^{-1}, $S_s = 10^{-5}$ m^{-1}, $H = 20$ m, $L = 50$ m, and $T_F = T = 0.002$ m^2 s^{-1} (the resulting c is 0.01 m^{-1}). The thick solid curve is calculated from the leaky fault solution given by (20); the thin solid curve is the Theis solution; and the dashed curve is the solution for a constant head along the fault. Note that drawdown for the leaky fault case is between that of the other two curves. Figure 3 shows the curves of equal drawdown in the pumped aquifer after 10 hours of pumping. The parameters used to calculate these equal drawdowns are the same as those used in preparing Figure 2. Because of leakage from the fault, the contours of equal drawdown (or the isopotentials) in the aquifer are noticeably deflected at the fault-aquifer intersection ($x = 0$).

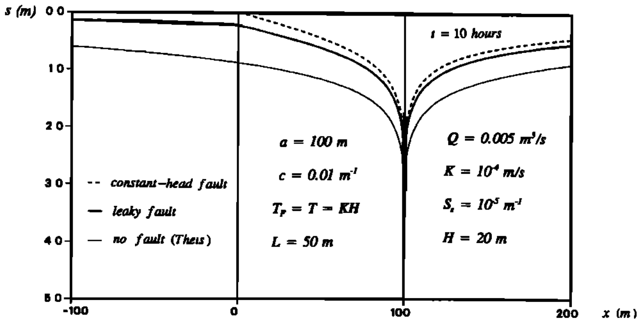

Figure 2. Aquifer depletion curves at the symmetry plane ($y = 0$): a comparison of the leaky fault solution with the constant head fault solution and the Theis solution.

The general solution (20) can also be used to estimate the recharge rate through the fault. As mentioned above, the second term in (7) is actually the recharge flux at an arbitrary value of y. If we integrate this term along the fault from $-\infty$ to $+\infty$ and take advantage of symmetry, we obtain

$$Q_r = 2\int_0^\infty \left(T_F\frac{s_2}{L}\right)_{x=0} dy = \frac{2T_F}{L}(I_1 - I_2) \tag{23}$$

where Q_r represents the overall recharge rate from the unpumped aquifer to the pumped aquifer moving through the fault; I_1 and I_2 are two integrals defined by

$$I_1 = \frac{Q}{4\pi T}\int_0^\infty W\left(\frac{a^2+y^2}{4\alpha t}\right) dy \tag{24}$$

$$I_2 = \frac{Qc\sqrt{\alpha}e^{ca}}{4\sqrt{\pi}T}\int_0^t \frac{e^{c^2\alpha\tau}}{\sqrt{\tau}}\operatorname{erfc}\left(\frac{a}{2\sqrt{\alpha\tau}} + c\sqrt{\alpha\tau}\right) d\tau \cdot \int_0^\infty e^{-y^2/(4\alpha\tau)}\,dy \tag{25}$$

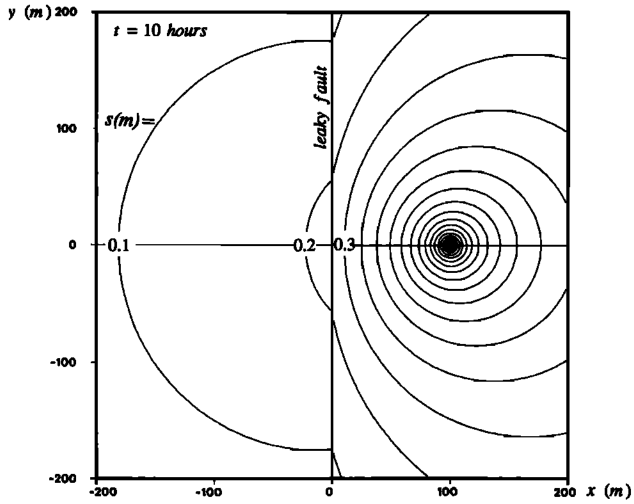

Figure 3. Drawdown in the pumped aquifer after 10 hours of pumping.

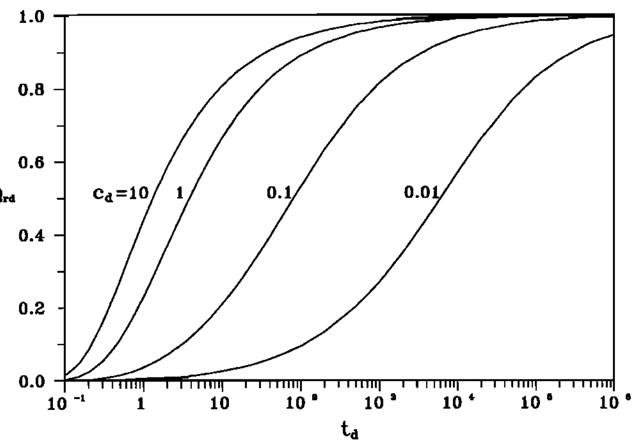

Figure 4. Effect of c_d on recharge rate through the fault to the pumped aquifer.

Noting that

$$\int_0^\infty \exp\left[-y^2/(4\alpha\tau)\right] dy = 2(\alpha\tau)^{1/2} \int_0^\infty \exp(-u^2)\, du$$

$$= (\pi\alpha\tau)^{1/2} \operatorname{erf}(\infty) = (\pi\alpha\tau)^{1/2} \tag{26}$$

the double integral (25) can be reduced to

$$I_2 = \frac{Qc\alpha e^{ca}}{4T} \int_0^t \exp(c^2\alpha\tau) \operatorname{erfc}\left(\frac{a}{2(\alpha\tau)^{1/2}} + c(\alpha\tau)^{1/2}\right) d\tau \tag{27}$$

Integrating by parts and interchanging the order of integration, the final solution for the overall recharge rate is

$$Q_r = Q\left[\operatorname{erfc}\left(\frac{a}{2\sqrt{\alpha t}}\right) - \exp(ca + c^2\alpha t) \cdot \operatorname{erfc}\left(\frac{a}{2(\alpha t)^{1/2}} + c(\alpha t)^{1/2}\right)\right] \tag{28}$$

which is a function of Q, α, t, a, and c. For recharge rate analysis we choose a as the characteristic length and introduce the following dimensionless variables and parameters:

$$Q_{rd} = \frac{Q_r}{Q} \qquad t_d = \frac{\alpha t}{a^2} \qquad c_d = c \cdot a \tag{29}$$

Applying (29) to (28), we obtain the dimensionless overall recharge rate

$$Q_{rd} = \operatorname{erfc}\left(\frac{1}{2\sqrt{t_d}}\right) - \exp(c_d + c_d^2 t_d) \cdot \operatorname{erfc}\left(\frac{1}{2(t_d)^{1/2}} + c_d(t_d)^{1/2}\right) \tag{30}$$

If $c_d = 0$ (corresponds to $T_F = 0$), then $Q_{rd} = 0$. For any $c_d \neq 0$, using L'Hospital's rule, the dimensionless overall recharge rate tends to 1 as t_d tends to infinity, which implies that the overall recharge rate will eventually increase to the limit of the pumping rate. For different c_d values, Figure 4 gives the time-dependent dimensionless overall recharge rate. For a given value of a, the dimensionless overall recharge rate increases to its limit more rapidly as c increases.

Case 2: Nonzero Drawdown in Unpumped Aquifer

In this case, drawdown in the unpumped aquifer is no longer zero, and we need to solve the problem by including the effects of drawdown in the unpumped aquifer. In the unpumped aquifer, by virtue of symmetry, only one half of the aquifer needs to be taken into consideration. We shall use the half corresponding to $x \le 0$ for convenience. Leakage through the fault is assumed to be proportional to the head difference (or, in other words, the drawdown difference) between pumped and unpumped aquifers. Based on these analyses the governing equations for the system are (1), (2), and

$$\frac{\partial s_u}{\partial t} = \alpha_u\left(\frac{\partial^2 s_u}{\partial x^2} + \frac{\partial^2 s_u}{\partial y^2}\right) \tag{31}$$

where we have used the subscript u for s and α to represent the drawdown and the diffusivity in the unpumped aquifer, respectively.

The initial and boundary conditions, (3) through (6), are the same as before except that corresponding conditions for s_u (the same as for s_2) should be added to (3) through (5). The boundary condition (7) needs to be modified to account for the drawdown in the unpumped aquifer which gives

$$T\frac{\partial s_1}{\partial x} = T\frac{\partial s_2}{\partial x} + T_F\frac{s_2 - s_u}{L} \qquad x = 0 \tag{32}$$

In addition, we have a flux boundary condition between the unpumped aquifer and the fault given by

$$T_F\frac{s_2 - s_u}{L} = 2T_u\frac{\partial s_u}{\partial x} \qquad x = 0 \tag{33}$$

where T_u represent the transmissivity of the unpumped aquifer; the factor 2 accounts for the symmetry of the unpumped aquifer about the fault.

Through the same operation procedure, that is, by taking the Laplace transform (12) and the Fourier transform (11), the governing equations for the pumped aquifer, (1) and (2), are reduced to (9) and (10), respectively; the governing equation for the unpumped aquifer (31) is reduced to

$$\frac{d^2 w_u}{dx^2} - A_u^2 w_u = 0 \tag{34}$$

where the definition for A_u is similar to that for A defined by (13)

$$A_u = (p/\alpha_u + \rho^2)^{1/2} \tag{35}$$

Using the same procedure given in the appendix, the solutions for (9), (10), and (34) that satisfy all boundary conditions are

$$w_1 = \frac{Q}{2T}\left[\frac{\exp(-|x-a|A)}{pA} - \frac{A_u \exp[-(x+a)A]}{pA(A/T_r + A_u + AA_u/c)}\right] \tag{36}$$

$$w_2 = \frac{Q}{2T} \cdot \frac{\exp[-(a-x)A](1/T_r + A_u/c)}{p(A/T_r + A_u + AA_u/c)} \tag{37}$$

$$w_u = \frac{Q}{2T} \cdot \frac{\exp(-aA + xA_u)/T_r}{p(A/T_r + A_u + AA_u/c)} \tag{38}$$

where T_r is the transmissivity ratio of the unpumped aquifer to the pumped aquifer, that is,

$$T_r = T_u/T \tag{39}$$

From the previous derivation the inversion for the first term in (36) is the Theis solution. The inversions for the other terms in (36) through (38) are developed for the following two possibilities.

1. Aquifers of equal diffusivity. For this special case we have $\alpha_u = \alpha$ and $A_u = A$. Substituting into (36) through (38) we obtain

$$w_1 = \frac{Q}{2T}\left[\frac{\exp(-|x-a|A)}{pA} - \frac{\exp[-(x+a)A]}{pA(A/c + 1/T_r + 1)}\right] \tag{40}$$

$$w_2 = \frac{Q}{2T}\left[\frac{\exp[-(a-x)A]}{pA} - \frac{\exp[-(a-x)A]}{pA(A/c + 1/T_r + 1)}\right] \tag{41}$$

$$w_u = \frac{Q}{2T}\cdot\frac{\exp[-(a-x)A]/T_r}{pA(A/c + 1/T_r + 1)} \tag{42}$$

The basic factor in the denominator of the above solution can be further decomposed as follows:

$$\frac{1}{A(A/c + 1/T_r + 1)} = \frac{c}{c'}\cdot\left(\frac{1}{A} - \frac{1}{A+c'}\right) \tag{43}$$

where c' is a constant defined by

$$c' = c(1/T_r + 1) = ck \tag{44}$$

The above decomposition is very convenient because one can use the previous results to obtain

$$s = \frac{Q}{4\pi T} W\left[\frac{(x-a)^2 + y^2}{4\alpha t}\right] - \frac{Qc\alpha^{1/2}\exp[c'(|x| + a)]}{4\pi^{1/2}T}\int_0^t g'(\tau)\,d\tau \tag{45}$$

$$s_u = \frac{1}{T_r}\cdot\frac{Qc\alpha^{1/2}\exp[c'(|x| + a)]}{4\pi^{1/2}T}\int_0^t g'(\tau)\,d\tau \tag{46}$$

where $g'(\tau)$ can be obtained by replacing c by c' in (21).

If the transmissivity of the unpumped aquifer is much larger than that of the pumped aquifer, or in other words, if the transmissivity ratio, T_r is very large, then the definition given by (44) can lead to $c' \approx c$. Under this condition, the drawdown in the unpumped aquifer will be negligibly small, and the solution for the pumped aquifer is reduced to solution (20) for case 1.

2. Aquifers with different diffusivities. For this more general problem it is difficult to derive exact inversions for (36) through (38). However, we can develop some approximate solutions for practical problems. For example, one can rewrite the common factor in (36) through (38) as

$$\frac{1}{A/T_r + A_u + AA_u/c} = \frac{c}{AA_u}\cdot\frac{1}{1+\varepsilon} \tag{47}$$

where the new parameter, ε, is defined as

$$\varepsilon = c\left(\frac{1}{A} + \frac{1}{T_rA_u}\right) \tag{48}$$

In practice, c is usually very small, and both A and A_u will have relatively large values at early time. This implies that we can assume $\varepsilon \ll 1$ such that $1/(1+\varepsilon) \approx 1 - \varepsilon$ becomes a good approximation for (47), which then simplifies (36) through (38) into

$$w_1 = \frac{Q}{2T}\left[\frac{\exp(-|x-a|A)}{pA} - \frac{c\exp[-(x+a)A]}{pA^2}\cdot\left(1 - \frac{c}{T_rA_u} - \frac{c}{A}\right)\right] \tag{49}$$

$$w_2 = \frac{Q}{2T}\left[\frac{\exp[-(a-x)A]}{pA} - \frac{c\exp[-(a-x)A]}{pA}\cdot\left(\frac{1}{A} + \frac{c}{T_rAA_u} + \frac{c}{T_r^2A_u^2}\right)\right] \tag{50}$$

$$w_u = \frac{Qc}{2TT_r}\cdot\frac{\exp(-aA + xA_u)}{pAA_u}\left(1 - \frac{c}{A} - \frac{c}{T_rA_u}\right) \tag{51}$$

The final solutions based on such an approximation are as follows:

$$s_1 = \frac{Q}{2T}\left[s^{(1)} - cs^{(2)} + \frac{c^2}{T_r}s^{(3)} + c^2s^{(4)}\right] \tag{52}$$

$$s_2 = \frac{Q}{2T}\left[s^{(1)} - cs^{(2)} - \frac{c^2}{T_r}s^{(3)} - \frac{c^2}{T_r^2}s^{(5)}\right] \tag{53}$$

$$s_u = \frac{Qc}{2TT_r}\left[s^{(6)} - cs^{(7)} - \frac{c}{T_r}s^{(8)}\right] \tag{54}$$

where

$$s^{(1)} = \frac{1}{2\pi}W\left[\frac{(x-a)^2 + y^2}{4\alpha t}\right] \tag{55}$$

$$s^{(2)} = \frac{\alpha^{1/2}}{2\sqrt{\pi}}\int_0^t \frac{\exp[-y^2/(4\alpha\tau)]}{\tau^{1/2}}\operatorname{erfc}\left(\frac{|x| + a}{2(\alpha\tau)^{1/2}}\right)d\tau \tag{56}$$

$$s^{(3)} = \frac{\alpha(\alpha_u)^{1/2}}{2\pi}\int_0^t d\tau\int_0^\tau \frac{\exp\{-y^2/[4(\alpha_u u + \alpha\tau - \alpha u)]\}}{\{u[\alpha_u u + \alpha(\tau - u)]\}^{1/2}}\cdot\operatorname{erfc}\left(\frac{|x| + a}{2[\alpha(\tau - u)]^{1/2}}\right)du \tag{57}$$

$$s^{(4)} = \frac{\alpha t}{\pi}\exp\{-[(x+a)^2 + y^2]/(4\alpha t)\} - \frac{(x+a)^2 + y^2}{4\pi}\cdot W\left[\frac{(x+a)^2 + y^2}{4\alpha t}\right] - \frac{\alpha^{1/2}(x+a)}{2\pi^{1/2}}\cdot\int_0^t \frac{\exp[-y^2/(4\alpha\tau)]}{\tau^{1/2}}\operatorname{erfc}\left(\frac{x+a}{2(\alpha\tau)^{1/2}}\right)d\tau \tag{58}$$

$$s^{(5)} = \frac{\alpha_u\alpha^{1/2}}{2\pi}\int_0^t d\tau\cdot\int_0^\tau \frac{\exp\{-y^2/[4(\alpha_u u + \alpha\tau - \alpha u)] - (a-x)^2/[4\alpha(\tau - u)]\}}{\{(\tau - u)[\alpha_u u + \alpha(\tau - u)]\}^{1/2}}du \tag{59}$$

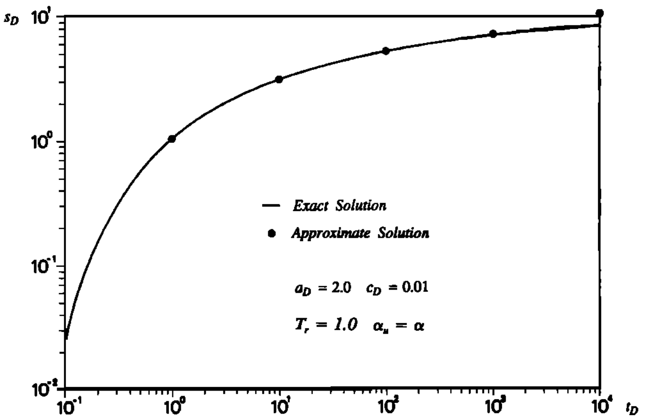

Figure 5. Evaluation of the approximate solution for aquifers of different diffusivity by setting $\alpha_u = \alpha$.

$$s^{(6)} = \frac{(\alpha\alpha_u)^{1/2}}{2\pi^{3/2}} \int_0^t d\tau$$

$$\cdot \int_0^\tau \frac{\exp\{-a^2/(4\alpha u) - x^2/[4\alpha_u(\tau - u)] - y^2/[4(\alpha u + \alpha_u\tau - \alpha_u u)]\}}{\{u(\tau - u)[\alpha u + \alpha_u(\tau - u)]\}^{1/2}} du \tag{60}$$

$$s^{(7)} = \frac{\alpha(\alpha_u)^{1/2}}{2\pi} \int_0^t d\tau$$

$$\cdot \int_0^\tau \frac{\exp\{-x^2/(4\alpha_u u) - y^2/[4(\alpha_u u + \alpha\tau - \alpha u)]\}}{\{u[\alpha_u u + \alpha(\tau - u)]\}^{1/2}}$$

$$\cdot \operatorname{erfc}\left(\frac{a}{2[\alpha(\tau - u)]^{1/2}}\right) du \tag{61}$$

$$s^{(8)} = \frac{\alpha_u \alpha^{1/2}}{2\pi} \int_0^t d\tau$$

$$\cdot \int_0^\tau \frac{\exp\{-a^2/(4\alpha u) - y^2/[4(\alpha u + \alpha_u\tau - \alpha_u u)]\}}{\{u[\alpha u + \alpha_u(\tau - u)]\}^{1/2}}$$

$$\cdot \operatorname{erfc}\left(\frac{|x|}{2[\alpha_u(\tau - u)]^{1/2}}\right) du \tag{62}$$

The details of this process as well as another approximate solution have been given by *Shan* [1990]. To see how good the approximation is, we compared the approximate solution (52) with the exact solution (45) by setting $\alpha_u = \alpha$. A point midway between the pumping well and the fault was chosen, and drawdowns in the pumped aquifer were calculated using the two solutions. The results are shown in Figure 5, where the calculated drawdowns from the exact solution are in excellent agreement with those from the approximate solution at early time (more discussion is given by *Shan* [1990]). For these calculations we used $T_r = 1$, and definitions for the other dimensionless parameters shown in the figure are given in the next section.

Fault Characterization

In calculating drawdown in the aquifer we have assumed that the hydraulic properties of both the aquifers and the fault were known. In practice, the hydraulic properties of the aquifers may have been determined by performing pumping tests at places far from the fault where the effect of fault leakage is insignificant. In addition, one will probably estimate the height of the fault, L, using geological data. What we really want is the transmissivity of the fault, T_F, which affects the solutions through the parameter c in (19).

Three sets of solutions have been derived under three different conditions: (1) drawdown in the unpumped aquifer is zero (equation (20)); (2) drawdown in the unpumped aquifer is nonzero while the two aquifers have the same diffusivity (equations (45) and (46)); and (3) drawdown in the unpumped aquifer is nonzero and the two aquifers have different diffusivities ((52) through (54)). In characterizing a leaky fault using a pumping test, it is always advantageous to have the observation well and the pumping well on the same side of the fault. Such a design is both convenient in field administration and favorable in obtaining a large drawdown at the observation well. Therefore we will focus on the problem of developing solutions in region 1 only. *Shan* [1990] has shown that the difference in diffusivity between two aquifers only affects drawdown significantly in the unpumped aquifer. We have assumed a diffusivity ratio of 100, and compared the drawdown in the pumped aquifer with that calculated from the equal-diffusivity solution. It was found that the difference between the two drawdown curves is negligibly small at early time (when $t_D < 10^4$). This is probably because the amount of water drained from the unpumped aquifer is mainly controlled by the relative transmissivities of the fault and the unpumped aquifer, particularly at early time. This result implies that even if the diffusivities of the two aquifers differ by two orders of magnitude, we may still use the equal-diffusivity solution (45) for the pumped aquifer to determine the transmissivity of the fault. Therefore we will only consider applications of the solutions under the first two conditions.

To determine c from pumping test data, let us first convert these solutions into a more convenient form by introducing

$$x - a = r \cdot \cos\theta \qquad y = r \cdot \sin\theta \tag{63}$$

$$s_D = \frac{4\pi T s}{Q} \qquad t_D = \frac{\alpha t}{r^2} \tag{64}$$

$$a_D = a/r \qquad c_D = c \cdot r \tag{65}$$

where r and θ are the polar coordinates of the observation well that is drilled in region 1. This time the radial distance, r, is used as the characteristic length instead of a in (29) to maintain a conventional definition for the dimensionless time, t_D. Using this transformation, we obtain the following dimensionless solutions in region 1: (1) Under condition 1,

$$s_D = W\left(\frac{1}{4t_D}\right) - \pi^{1/2}c_D \exp\left[c_D(2a_D + \cos\theta)\right] \int_0^{t_D} G(\tau)\, d\tau \tag{66}$$

where

$$G(\tau) = \frac{\exp\left[c_D^2\tau - \sin^2\theta/(4\tau)\right]}{\tau^{1/2}} \operatorname{erfc}\left(\frac{2a_D + \cos\theta}{2\tau^{1/2}} + c_D\tau^{1/2}\right) \tag{67}$$

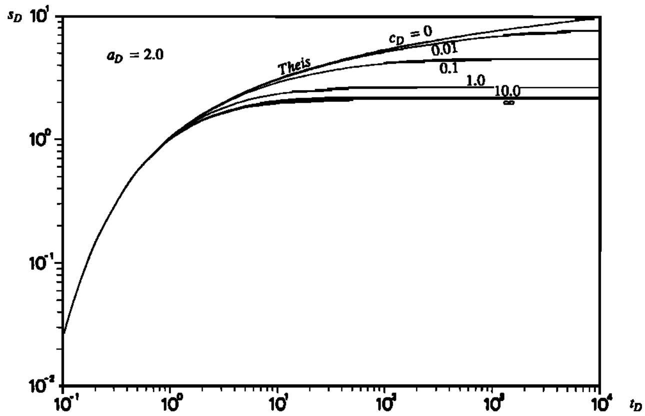

Figure 6. Type curves for six values of c_D at the point of $r = a/2$ and $\theta = \pi$.

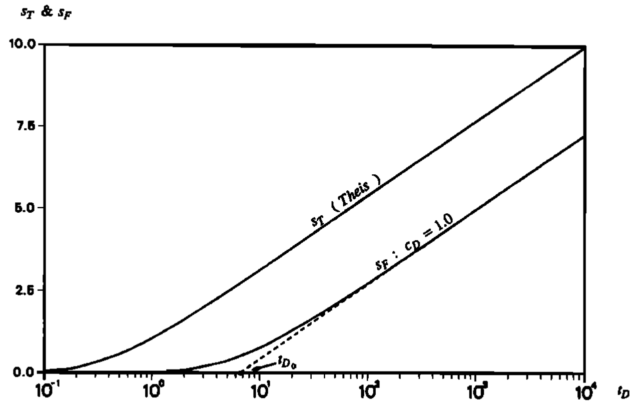

Figure 7. Dimensionless time intersect, t_{D0}, at the point of $r = a/2$ and $\theta = \pi$.

(2) Under condition 2 and in the pumped aquifer,

$$s_D = W\left(\frac{1}{4t_D}\right) - \frac{\pi^{1/2}}{k} c'_D \cdot \exp\left[c'_D(2a_D + \cos\theta)\right] \int_0^{t_D} G'(\tau)\, d\tau \qquad (68)$$

where c'_D is defined by (65) using c'; c' and k are defined by (44); and $G'(\tau)$ can be obtained by replacing c_D by c'_D in (67).

In practice, one should install observation wells in both aquifers. The drawdown data obtained from the observation well screened in the unpumped aquifer will help to determine which solution is appropriate. For example, if there is no measurable drawdown in this observation well, or the drawdown is negligibly small, one may use (66) and the drawdown data from the pumped aquifer to determine the hydraulic properties of the fault.

For convenience one may choose the observation well in the pumped aquifer at a point midway between the pumping well and the fault, such that $r = a/2$ and $\theta = \pi$. From (65) we know that $a_D = 2$. Using different c_D values, a set of s_D-versus-t_D-type curves can be calculated (Figure 6). The observed drawdown data for s and t can be easily converted to dimensionless s_D and t_D data using (64). By plotting these data in Figure 6, one can estimate the value of c_D by curve matching. A semilog method can also be developed. The dimensionless drawdown, s_D in (66), is composed of two terms: s_T and s_F. If both components are plotted in a semilog manner, as shown on Figure 7, we find that both curves become straight as time increases. Data for s_F can be obtained by subtracting the observed result for s_D from the calculated value of s_T. The time intercept, t_{D0} in Figure 7, can then be used to determine the objective parameter, c_D. Based on a series expansion of the complementary error function in the integrand in (67), *Shan* [1990] derived the following approximate formula at the point $r = a/2$ and $\theta = \pi$:

$$\exp(1.5c_D)W(1.5c_D) = \ln t_{D0} - 1.3872 \qquad c_D > 0.17 \qquad (69)$$

The condition given in the formula is necessary to guarantee convergence of the series. With this formula one can always calculate c_D using the iteration method.

Since (68) is similar to (66), the application to problems under the second condition is the same as that under the first condition described above.

Summary

We have analyzed the problem of saturated water flow in an aquifer-fault-aquifer system subject to a single-well pumping test. Analytical solutions have been developed for three different cases: (1) drawdown in the unpumped aquifer is negligibly small; (2) drawdown in the unpumped aquifer is significant, and the two aquifers have the same diffusivity; and (3) drawdown in the unpumped aquifer is significant, and the two aquifers have different diffusivities. These solutions were derived based on the following assumptions: (1) both aquifers are homogeneous, isotropic, and horizontally infinite with uniform thickness; (2) the fault connecting the two aquifers is homogeneous, vertical (or near vertical), and semi-infinite; (3) the compressive storage of the fault is negligible, and the recharge rate through the fault is proportional to the vertical head drop in the fault. An approximate solution was obtained for the third case using a truncated series expansion. An analysis of this approximate solution indicates that at early time, drawdown in the pumped aquifer is insensitive to the difference in diffusivity between the two aquifers. This feature permits one to apply the solution of case 2 to some case 3 problems. The results of this investigation can be used to predict the drawdown in the aquifer(s), estimate the recharge rate through the leaky fault, and, more importantly, determine the transmissivity of the fault. Examples have been included to show the potential applications of these solutions to practical problems.

Appendix

The general solutions to (9) and (10) are

$$w_1 = \left[C_1 - \frac{Q\exp(-Aa)}{2pTA}u(x-a)\right]\exp(A_x) + \left[C_2 + \frac{Q\exp(Aa)}{2pTA}u(x-a)\right]\exp(-Ax) \qquad (A1)$$

$$w_2 = C_3\exp(Ax) + C_4\exp(-Ax) \qquad (A2)$$

where C_1, C_2, C_3, and C_4 are four integral constants. In deriving (A1) we have used the formula

$$\int_0^x f(y)\delta(y-a)\,dy = f(a)\cdot u(x-a) \qquad (A3)$$

where $u(x - a)$ is the unit step function which equals 0 for $x < a$ and 1 for $x \geq a$.

To satisfy (14), we must have

$$C_1 = \frac{Q \exp(-Aa)}{2pTA} \quad \text{(A4)}$$

$$C_4 = 0 \quad \text{(A5)}$$

To satisfy (15) and (16), we must have

$$C_1 + C_2 = C_3 \quad \text{(A6)}$$

$$TA(C_1 - C_2) = TAC_3 + T_F C_3 / L \quad \text{(A7)}$$

Solving (A6) and (A7) simultaneously, we obtain

$$C_2 = \frac{-c}{A + c} C_1 = \frac{-Qc \exp(-Aa)}{2pTA(A + c)} \quad \text{(A8)}$$

and

$$C_3 = \frac{A}{A + c} C_1 = \frac{Q \exp(-Aa)}{2pT(A + c)} \quad \text{(A9)}$$

where the newly introduced parameter, c, is defined by (19) and represents the combined effect of the relative transmissivity of the leaky fault, T_F/T, and its path length, L.

Substituting the four integral constants into (A1) and (A2) and applying the definition for the unit step function, we obtain (17) and (18), which are the solutions in the transformed domain. To obtain their inversions, let us first decompose (17) into

$$w_1 = \frac{Q}{2T} \left[\frac{\exp(-|x - a|A)}{pA} - \frac{\exp[-(x + a)A]}{pA} + \frac{\exp[-(x + a)A]}{p(A + c)} \right] \quad \text{(A10)}$$

The second term is similar to the first term in (A10), while the third term in (A10) is similar to the term in (18), which means that we need to invert only two kinds of terms, described as follows:

$$\mathcal{F}^{-1}\left\{ \mathcal{L}^{-1}\left\{ \frac{\exp[-(x + a)A]}{pA} \right\} \right\} = \frac{1}{2\pi} W\left[\frac{(x + a)^2 + y^2}{4\alpha t} \right] \quad \text{(A11)}$$

$$\mathcal{F}^{-1}\left\{ \mathcal{L}^{-1}\left\{ \frac{\exp[-(x + a)A]}{p(A + c)} \right\} \right\} = \frac{1}{2\pi} W\left[\frac{(x + a)^2 + y^2}{4\alpha t} \right] - \frac{c\alpha^{1/2} e^{c(x+a)}}{2\pi^{1/2}} \int_0^t g(\tau)\, d\tau \quad \text{(A12)}$$

where $W(u)$ is the well function which is defined by (22), and $g(\tau)$ is defined by (21). In deriving (A11) and (A12) we have used the following inversion formulae [*Erdélyi*, 1954]:

$$\mathcal{L}^{-1}\{f(p + b)\} = e^{-bt} \cdot \mathcal{L}^{-1}\{f(p)\} \quad \text{(A13)}$$

$$\mathcal{L}^{-1}\{f(p)/p\} = \int_0^t \mathcal{L}^{-1}\{f(p)\}\, dt \quad \text{(A14)}$$

$$\mathcal{L}^{-1}\left\{ \frac{\exp(-bp^{1/2})}{p^{1/2} + d} \right\} = \frac{\exp[-b^2/(4t)]}{(\pi t)^{1/2}} - d \exp(bd + d^2 t) \operatorname{erfc}\left(\frac{b}{2t^{1/2}} + dt^{1/2} \right) \quad \text{(A15)}$$

$$\mathcal{F}_c\{\exp(-b\rho^2)\} = \frac{\pi^{1/2}}{2b^{1/2}} \exp[-y^2/(4b)] \quad \text{(A16)}$$

where erfc(x) is the complementary error function, and $\mathcal{F}_c$ stands for the Fourier cosine transform that has the following relationship with the inverse Fourier exponential transform:

$$\mathcal{F}^{-1}\{w(\rho^2)\} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} w(\rho^2) \exp(-i\rho y)\, d\rho = \frac{1}{\pi} \int_0^{+\infty} w(\rho^2) \cos(\rho y)\, d\rho = \frac{1}{\pi} \mathcal{F}_c\{w(\rho^2)\} \quad \text{(A17)}$$

(Note that since $w(\rho^2) \sin(\rho y)$ is an odd function of ρ, its integration with respect to ρ from $-\infty$ to $+\infty$ is definitely zero.)

Applying (A11) and (A12) to (A10) and (18), and comparing the solutions in the two regions, we obtain the unified expression, (20) for the drawdown at any point in the aquifer.

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I. Javandel, C. Shan, and P. A. Witherspoon, Earth Sciences Division, Lawrence Berkeley Lab., University of California, Bldg. 50E, Berkeley, CA 94720.

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