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A METHOD OF ANALYZING PUMP TEST DATA FROM FRACTURED AQUIFERS

Méthode en vue d'évaluer les essais de pompage dans des conduits d'eau fissurés

Eine Methode zur Auswertung von Pumpversuchen in klüftigen Wasserleitern

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SUMMARY

In the field of groundwater hydrology, the traditional method of analyzing the non-steady behavior of a confined aquifer during a pumping test relies on the Theis solution for radial flow in homogeneous, isotropic and infinite system. By an appropriate graphical matching of field data with the type curve based on the Theis solution, it is possible to determine both the permeability and the storage coefficient for the aquifer. The computed results are affected by departures from the idealized conditions assumed by Theis.

When fractures are intersected by the pumping well, the non-steady behavior of the aquifer differs in significant ways from that predicted by the Theis solution, and a different approach is required. Analytical solutions for flow to a well that intersects a single fracture, either horizontal or vertical, are now available, from which type curves have been developed. With the aid of these type curves, it is possible to distinguish between aquifers with horizontal and vertical fractures, and to analyze the system as an "equivalent" anisotropic homogeneous porous medium with a single fracture of much higher permeability. It can be shown that use of the Theis solution in analyzing such systems can lead to significant errors. Field examples are included to demonstrate how the new type curves can be used to determine the nature, extent and direction of the single fracture in the equivalent porous medium.

INTRODUCTION

Well established procedures in the field of groundwater hydrology now make it possible to determine the hydraulic properties of confined aquifers. The analysis is usually based on the Theis solution, which assumes an isotropic, homogeneous aquifer of infinite radial extent. Unfortunately, the same method of analysis is often used when the aquifer does not satisfy these idealized conditions.

This can be a real source of difficulty if one is dealing with naturally fractured aquifers. Field data are available which indicate that the flow behavior of fractured systems can be significantly different from that of homogeneous systems [Lewis and Burgy, 1964]. A different solution is thus required to analyze such fractured systems. This is especially critical if fluid level measurements are only taken at the pumping well. Commonly, a log-log plot of drawdown versus pumping time yields an initial straight line with a slope of 0.5 that, according to the literature, has no counterpart in any mathematical model for fractured reservoirs. These early time data are generally not considered for analysis.

A new mathematical model is presented here, which shows that the early straight line is due to a fracture intersecting the pumping well. This model can be used to analyze such a system in terms of an "equivalent", anisotropic porous medium with a single fracture of much higher permeability that intersects the pumping well. In most cases, it is possible to determine the nature of the fracture system from drawdown data as measured at the pumping well. For systems with vertical fractures, the analysis also yields the geometric mean of the maximum and minimum anisotropic permeabilities, $\sqrt{K_x K_y}$. If fluid levels are also measured at observation wells, the permeabilities K_x and K_y, and the length and direction of the fracture intersecting the well can be computed. For systems with horizontal fractures, the vertical and horizontal permeabilities, as well as the radius of the fracture intersecting the pumpint well can be obtained in some cases using only data that have been measured in the pumping well.

PREVIOUS WORK

A number of mathematical models have been proposed in the past to explain the observed flow behavior of fractured systems. Two concepts have been advanced, viz., that of an equivalent double-porosity system, and that of an equivalent anisotropic reservoir.

Double porosity model (Figure 1)

This model assumes that two regions of distinctly different porosity exist within the formation: one region, the matrix, has a high storage and low flow capacity, while the other, the fracture, has a low storage and high flow capacity. The primary, or matrix, porosity is homogeneous and isotropic, and is contained within an array of identical parallel-

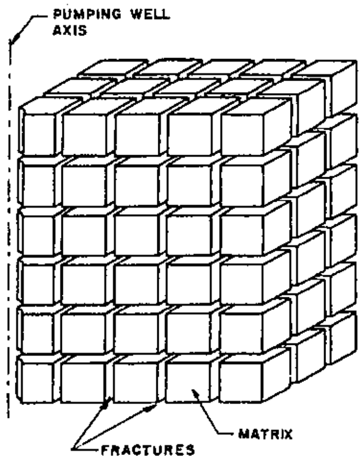

Fig. 1. Double porosity model for fractured aquifers [Warren and Root, 1963]

epipeds. The secondary porosity is contained within an orthogonal system of continuous, uniform fractures of uniform anisotropic permeability. Flow exists between the primary and secondary porosities, but it is assumed that the matrix permeability is so small that movement of fluid to the pumping wells occurs only in the fracture system [Barenblatt, et al., 1960].

Variations of this model have been investigated by a number of authors in the field of petroleum engineering, who have been concerned with the problem of data as measured at the producing well [Pollard, 1959; Warren and Root, 1963; Odeh, 1965; Kazemi, 1969]. In general, the conclusions reached by different workers have not been in agreement (see Matthews and Russell, 1967, and Kazemi, 1969, for a detailed review of this work). It appears that different results are possible for fractured systems, depending upon the reservoir characteristics encountered. For example, a fractured aquifer is equivalent to an homogeneous porous medium if the dimensions of the matrix blocks are small (less than 3 feet), and the matrix permeabilities are significant (greater than 10^{-2} md or 10^{-8} cm/sec).

The double-porosity model has also been used to interpret interference test data, i.e., pressure changes as measured in observation wells at some known distance from the pumping well. It was found that only under certain conditions and at large values of time could a fractured system be considered equivalent to a homogeneous system [Kazemi, et al., 1969; Rofail, 1965].

Anisotropic models (Figure 2)

A different approach to the problem of flow in fractured systems is to analyze a fractured reservoir as an equivalent anisotropic system. An example of such an analysis for pressure drawdowns as measured at the producing well was presented by Adams, et al., (1968). The authors noticed a peculiar behavior of a low permeability, fractured dolomite gas reservoir that could be interpreted as an increase in permeability at some distance from the well. Unlike the previous double porosity model, the well could communicate with the fracture system via the relatively tight matrix. The fracture reservoir was thus analyzed by means of an equivalent "composite" reservoir, i.e., the system consists of two concentric, homogeneous and isotropic, annular regions of distinctly different permeability (Figure 2a).

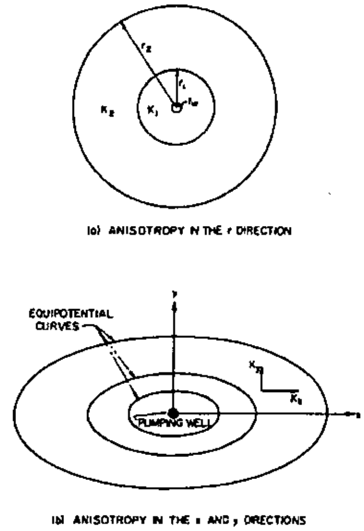

Fig. 2. Anisotropic models for fractured reservoirs

The anisotropic effect of a fractured system may be easier to visualize in the case of a vertical fracture system. Experiments conducted by Huskey and Crawford (1967) indicate that permeability should increase when the fractures are parallel to the direction of flow, whereas there should be no change in permeability due to fractures at right angles to the direction of flow. As a result, the maximum permeability should be representative of the fracture system, while the minimum permeability will reflect the effects of the matrix (see Figure 2b).

This concept was used by Elkins and Skov (1960) to analyze pressure transients observed during the development of the Spraberry field, which is a low permeability oil reservoir (1 md or 10^{-6} cm/sec) that is cut by an extensive system of vertical fractures. The authors employed an equation developed by Collins (1961) for the pressure drawdown at the location of one well, due to constant production at another well in an extensive anisotropic reservoir of uniform thickness. The method, however, required a knowledge of the principal axes of permeability, which in this case, were along and at right angles to the major fracture trend. A more general equation for the drawdown has been derived by Papadopulos (1965) with respect to an arbitrary system of axes. He also developed a method for finding the hydraulic properties of the system from pump test data, and the directions of maximum and minimum permeabilities.

A similar method of analysis was proposed by Saad (1967) for investigating reservoirs with natural horizontal fractures. The technique suggested would allow computation of the horizontal and vertical permeabilities, provided that the pumping well partially penetrates the aquifer [Hantush, 1957]. However, these methods cannot be used to analyze systems where data are taken only at the pumping well.

The equivalence between naturally fractured porous rock and anisotropic homogeneous media was further documented by Parsons (1966), and Kiràly (1971). Parsons indicated that the property of equivalence would hold only for fracture distributions that are not too disperse. On the other hand, if the system is highly fractured, the pumping well is likely to intersect at least one of the fractures, and this affects the unsteady flow behavior of the reservoir in a characteristic way. This case can be analyzed with the techniques to be discussed below.

MATHEMATICAL MODELS

The mathematical models presented here were originally developed for the special situation of a hydraulically fractured aquifer. Analytical solutions were developed to describe the pressure behavior of a well that intersects a single horizontal fracture [Gringarten and Ramey, 1971a] or a single vertical fracture [Gringarten, et al., 1972] while the well produces at constant rate from an otherwise homogeneous porous medium. Two types of solutions were obtained. One solution was based on the assumption of a uniform head along the fracture (i.e., a fracture of infinite conductivity), and the results of this analysis match field data from typical hydraulic fracturing experience. The other solution assumed that fluid enters the fracture at the same rate per unit area (i.e., a fracture with uniform flux) and the results seem to match the behavior of a well that intersects a natural fracture. Inspection of a number of pump test results indicates that the latter solution should be useful in evaluating data from fractured confined aquifers.

One important finding of this work was the interesting observation that for both solutions, a log-log plot of fluid level drawdowns versus producing time yields a characteristic straight line with a slope of 0.5 at early times, indicating linear flow from the reservoir matrix into the fracture. At long times, the drawdown behavior is the same as that indicated by the Theis solution plus a constant that depends on the point where measurements are being made.

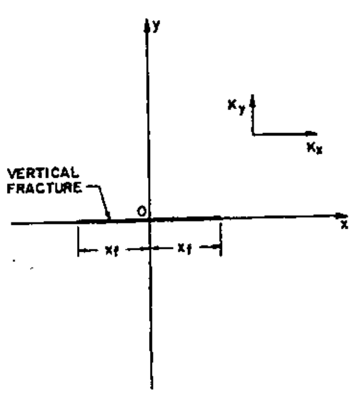

Fig. 3. Vertical fracture model

Vertical fracture model

The Gringarten, et al. (1972) solution for a well intersecting a single plane vertical fracture in a radially infinite porous medium has been modified to account for anisotropy. A plan view of the two-dimensional model is sketched in Figure 3. The permeability K_x in the x direction is parallel to the fracture and can be interpreted as representing the increased permeability due to the existence of a major trend of vertical fractures in the x direction. K_y represents the matrix permeability. The pumping well is located along the vertical fracture axis.

The pressure drawdown, obtained by means of the Green's function and product solution method [Gringarten and Ramey, 1971b] can be expressed as:

$$s_D = \frac{\sqrt{\tau}}{2} \int_0^{t_D} \left[\operatorname{erf} \frac{1 - \frac{x}{x_f}}{2\sqrt{\tau}} + \operatorname{erf} \frac{1 + \frac{x}{x_f}}{2\sqrt{\tau}} \right] \frac{e^{-\frac{1}{4}\left(\frac{y}{x_y}\right)^2 \frac{T_x}{T_y}}}{\sqrt{\tau}} \, d\tau \tag{1}$$

where

$$t_D = \frac{K_x t}{S_s x_f^2} = \frac{T_x t}{S \, x_f^2} \tag{2}$$

$$s_D = \frac{4\pi \sqrt{K_x K_y} \, Hs}{Q} = \frac{4\pi \sqrt{T_x T_y} \, s}{Q} \tag{3}$$

x_f represents the fracture half length. At long times, the drawdown can be approximated by:

$$s = \frac{2.303\, Q}{4\pi\sqrt{T_x T_y}} \log_{10} (2.25\, t_D') + \frac{Q}{2\sqrt{T_x T_y}} \left\{ 1 - \frac{2.303x}{4x_f} \log_{10} \frac{T_x y^2 + (x+x_f)^2 T_y}{T_x y^2 + (x-x_f)^2 T_y} - \frac{y\sqrt{T_x T_y}}{2x_f} \arctan \frac{.yx_f\sqrt{T_x T_y}}{T_x y^2 + (x-x_f)^2 T_y} \right\} \tag{4}$$

where

$$t_D' = \frac{t}{S} \frac{T_x T_y}{[T_x y^2 + (x+x_f)^2 T_y]^{1/2} [T_x y^2 + (x-x_f)^2 T_y]^{1/2}}$$

Eqs. 1 and 4 are the basis for analyzing pump test data.

The dimensionless drawdown at the pumping well (x=y=0) is obtained from Eq. 1 as:

$$s_D = 2\pi\sqrt{t_D} \operatorname{erf} \left(\frac{1}{2\sqrt{t_D}}\right) - \operatorname{Ei} \left(-\frac{1}{4t_D}\right) \tag{5}$$

A log-log plot of s_D versus t_D from Eq. 5 is shown in Figure 4. This plot is characterized by an initial half unit slope, straight line and can be used as a type curve in analyzing drawdown data that exhibit the same characteristics. Customarily, drawdowns observed at the pumping well are plotted versus time on logarithmic paper of the same scale and then matched with the type curve. From the dual coordinates of an arbitrary matching point, (s,t) and (s_D, t_D) one obtains the relations:

$$\sqrt{T_x T_y} = \frac{Q}{4\pi} \frac{s_D}{s} \tag{6}$$

$$\frac{T_x}{S\, x_f^{\,2}} = \frac{t_D}{t} \tag{7}$$

The first equation yields the geometric mean of the maximum and minimum transmissibilities. The second equation can be solved for

$$\frac{T_x}{x_f^{\,2}}$$

if a value for storage is available. If the aquifer is an indurated rock and porosities have been measured, the storage can be estimated from $S = \gamma\phi cH$.

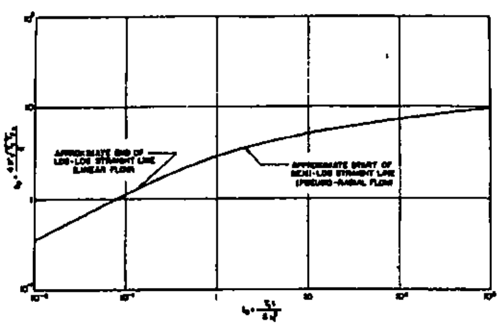

Fig. 4. Type curve for vertical fracture model with drawdown measured at pumping well.

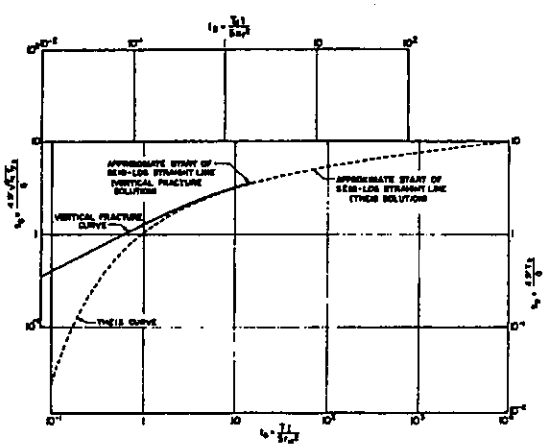

Fig. 5. Comparison between Theis and vertical fracture solution.

A measurement of the error involved in analyzing data from a vertical fracture system with the Theis solution can be obtained by matching the type curve from Figure 4 with the Theis solution, as shown in Figure 5. If early time data from the fractured system are neglected, a good match can be obtained with the Theis solution. It should be noted that the scale on the drawdown axis is identical for both curves, whereas the scale on the time axis differs by more than a log-cycle. In other words, if one were to analyze field data from a fractured aquifer by matching with the Theis solution, the analysis would yield an apparent permeability:

$$T' = \frac{Q}{4\pi} \frac{s_D}{s} \tag{8}$$

from which an apparent storage could be calculated using:

$$S' = \frac{T'}{t_D'} \frac{t}{r_w^{\,2}} \tag{9}$$

where s_D has the same value as in equation 6, $t_D' = 7.15\ t_D$ and r_w is the radius of the pumping well. Inspection of equations 6 and 8, and equations 7 and 9 shows that:

$$T' = \sqrt{T_x T_y} \tag{10}$$

$$S' = \frac{1}{7.15} \frac{T'}{T_x} \left(\frac{x_f}{r_w}\right)^2 S = \frac{1}{7.15}\sqrt{\frac{T_y}{T_x}} \left(\frac{x_f}{r_w}\right)^2 S \tag{11}$$

Therefore, the isotropic transmissibility coefficient obtained when using the Theis solution would in fact represent the geometric mean of the anisotropic transmissibilities. Furthermore, as shown by equation 11, computation of S using equation 9 would yield a much higher value than the real one obtained from equation 7, because the magnitude of

$$\frac{x_f}{r_w}$$

would tend to dominate.

As indicated by equation 4 for large values of time, the analysis can also be performed by means of the "straight line" method. The time at which the semi-log straight line begins is indicated by an arrow in Figure 4.

Since the hydraulic properties of the system cannot be individually determined from the above analysis, it is apparent that observations at other points in the system are needed. Drawdown measurements at a minimum of two observation wells are required to provide the necessary data.

The transient response at observation wells differs from that at the pumping well (see Figure 6). No initial half-unit slope straight line is observed on log-log coordinates, except in the case where the observation well intersects the same vertical fracture as the pumping well (Figure 6c). The observed pressure response depends upon the location of the observation well, and the transmissibility ratio, which makes the "type curve matching" method difficult to use. However, far enough from the pumping well,

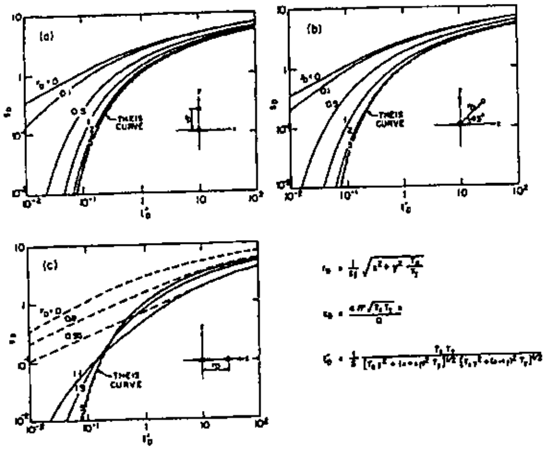

Fig. 6. Type curves for vertical fracture model with drawdowns measured at observation wells.

the behavior of the system becomes identical to that of a line source well in an anisotropic porous medium. This is apparent for $r_D > 5$ in Figure 6, and can be verified from equation 2. The second term on the right hand side of equation 4 vanishes as:

$$r_D = \frac{1}{x_f}\sqrt{x^2 + y^2\frac{T_x}{T_y}}$$

increases, or x_f decreases, while the first term becomes identical to the long time approximation for the drawdown created by a line source well in an anisotropic aquifer, written with respect to the principal axes of transmissibility (Collins, 1961; Papadopulos, 1965). This is realized within 1% when:

$$x^2 + y^2\frac{T_x}{T_y} > 25\ x_f^{\,2} \tag{12}$$

In such a case, methods described by Papadopulos (1965) can be used for analysis.

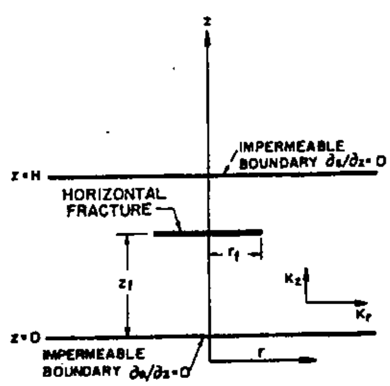

Fig. 7. Horizontal fracture model.

Horizontal fracture model

A graphical sketch of a horizontal fracture model is given in Figure 7. The pumping well is located along the axis of the plane horizontal fracture. The permeability K_r in the radial direction is parallel to the fracture and can be interpreted as representing the increased permeability due to the existence of a horizontal fracture system. The permeability K_z in the vertical direction represents the matrix permeability.

An analytical solution for this case, as obtained by Gringarten and Ramey (1971a), can be written as:

$$s_D = \frac{2}{H_D}\int_0^{t_D}\frac{e^{-\frac{1}{4\tau}\left(\frac{r}{r_f}\right)^2}}{\tau}\left[\int_0^1 I_o\left(\frac{r\nu}{2r_f\tau}\right)e^{-\frac{\nu^2}{4\tau}}\,\nu d\nu\right]\left[1 + 2\sum_{n=1}^{\infty}\exp\left(-\frac{n^2\pi^2\tau}{H_D^{\,2}}\right)\cos n\pi\frac{z_f}{H}\cos n\pi\frac{z}{H}\right]d\tau \tag{13}$$

where:

$$t_D = \frac{K_r t}{S_s r_f^{\,2}} \tag{14}$$

$$s_D = \frac{4\pi\sqrt{K_r K_z}\ r_f s}{Q} \tag{15}$$

$$H_D = \frac{H}{r_f}\sqrt{\frac{K_r}{K_z}} \tag{16}$$

and r_f is the radius of the horizontal fracture.

As in the vertical fracture case, type curve matching or the straight line method can be used for analysis. Type curves for analyzing drawdowns at the pumping well ($r=0$, $z=z_f$) when the fracture is at the center of the formation are presented in Figure 8. A log-log plot of s_D versus t_D is given for various values of dimensionless thickness, H_D. It should be

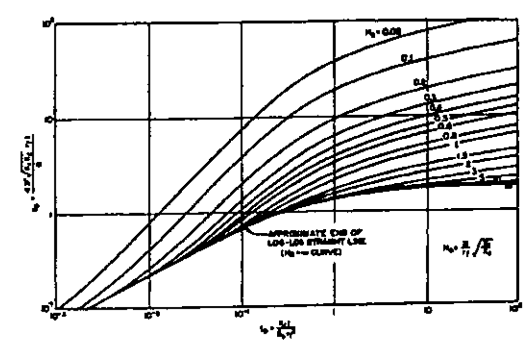

Fig. 8. Type curves for horizontal fracture model with drawdowns measured at the pumping well.

noted that the definitions of s_D and t_D are not the same as in the case of the vertical fracture model. One sees again that at early times, a straight line with a half-unit slope is obtained. Aside from this similarity, the shapes of the horizontal fracture curves on Figure 8 are sufficiently different from that of the vertical fracture curve on Figure 4 (except for $1<H_D<3$) so that it should be possible to distinguish between the two types in a pumping test. On the other hand, the horizontal fracture curves corresponding to $1<H_D<3$ are very similar to the vertical fracture curve on Figure 4, and caution should be exercised in interpreting field data for these special cases.

By matching drawdown data with the type curves on Figure 8, one can obtain three equations:

$$\sqrt{K_r K_z}\, r_f = \frac{Q\, s_D}{4\pi H s} \quad (17)$$

$$\frac{K_r}{S_s r_f^2} = \frac{t_D}{t} \quad (18)$$

$$H_D = \frac{H}{r_f}\sqrt{\frac{K_r}{K_z}} \quad (19)$$

from which K_r, K_z and r_f can be calculated if a value for S_s is available or can be estimated.

In some cases, however, H is so large that the data plot follows the type curve $H_D = \infty$ over the entire test period. In this event, it is not possible to determine K_r, K_z and r_f from data measured only in the pumping well. The straight line method will not help either because the time at which the semilog straight line begins is given by:

$$t = \sup \left[\frac{S_s H^2}{2K_z};\ 12.5\ \frac{S_s(r^2 + r_f^2)}{K_r}\right] \quad (20)^*$$

and may be very large if H is large.

As in the vertical fracture case, field data from the pumping well can be matched to the Theis solution if early time data are neglected. It is obvious that erroneous results should then be expected.

Drawdown behavior at observation wells in a horizontally fractured system depends upon the distance to the pumping well (Gringarten and Ramey, 1971a). If the observation well intersects the fracture, an initial half unit slope straight line appears on a log-log plot. If the distance to the pumping well is greater than the fracture radius, this is no longer so, and there exists a critical distance from the pumping well, equal to:

$$r_1 = r_f + 2H\sqrt{\frac{K_r}{K_z}} \quad (21)$$

beyond which the drawdown is the same as that created by a line source well in a homogeneous reservoir with isotropic radial permeability, K_r. Drawdown data from such observation wells can be analyzed with the Theis solution to provide values of K_r and S_s.

DISCUSSIONS AND CONCLUSIONS

We hope it is clear that indiscriminate use of the Theis solution in analyzing pump test data from fractured aquifers can yield grossly erroneous values for the hydraulic parameters of the system. When the system is known to be fractured, we believe alternative solutions to the Theis equation should be considered. The models discussed above should be of some assistance.

The method presented here provides a way of analysing fractured aquifers as an "equivalent" anisotropic, homogeneous porous medium, with a single fracture of much higher permeability intersecting the pumping well. It should be applied to those field cases where a log-log plot of drawdown versus pumping time, as observed at the pumping well, reveals an initial half unit slope, straight line. By matching the data plot with the type curves presented in this paper, it should be possible to determine whether the fracture trend is essentially horizontal, vertical, or neither of these. In those cases where a match can be obtained, the hydraulic properties of the system can be computed as outlined above.

Recovery data can be used for analysis, instead of drawdown data. A log-log plot of water level recovery versus time since pumping stopped has the same shape as that obtained with drawdown data if the recovery time is small compared to the pumping time (Ramey, 1970). The aquifer should also be of infinite extent, or bounded by an impermeable or zero drawdown outer boundary. Recovery data can also be used when the conditions at the outer boundary are that of prescribed flux, or drawdown, if the pumping time has been small enough so that no boundary effects can be detected on the drawdown curves.

The models presented here should also be applicable to nonporous systems such as fractured cristalline rock for which an initial straight line of half-unit slope on log-log paper is apparent from field data as measured in the pumping well. In such cases, however, the specific storage S_s used in the formula would represent the storage factor of an equivalent porous medium. If possible, this value should be confirmed by analyzing data that have been measured at observation wells.

An example is shown in Figure 9. The water level recovery as measured at an observation well in a finely fractured cristalline rock has been plotted versus time since pumping stopped [Marine, 1967]. The author determined hydraulic constants from type curve matching with the Theis solution (Figure 9a), on the assumption that the fractures were so uniformly distributed that the idealized conditions of homogeneity and anisotropy are reasonably well satisfied. However, another interpretation is also possible. As shown on Figure 9b, a straight line of slope 0.5 can be drawn through the first six data points. This interpretation and the reservoir configuration as given by Marine [1967], which is reproduced on Figure 9c, suggest that the horizontal fracture solution could be applied to this particular situation.

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* sup means "use whichever term is larger".

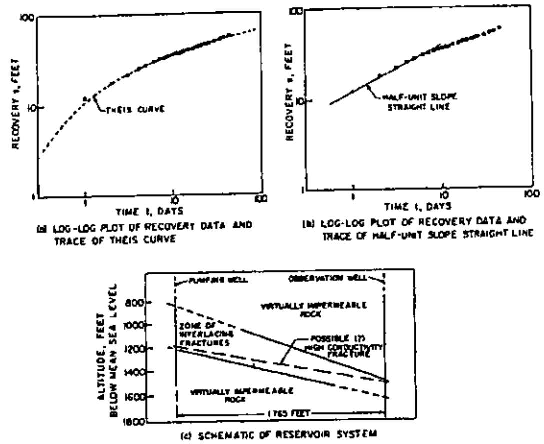

Fig. 9. Analysis of recovery data from fractured cristalline rock.

In some cases, the initial half unit slope, straight line is not apparent, and a unit slope straight line is obtained instead. This indicates the effect of a large storage volume connected to the pumping well, and corresponds to a fracture of large dimensions rather than a planar fracture as used in this paper (Ramey, 1970; Gringarten and Ramey, 1971a). It is normally still possible to recognize the half-unit slope, straight line and to analyze the system with the type-curves given in this paper.

Finally, it must be recognized that the method presented here may not be the unique answer to the problem, and one must consider other possible explanations before reaching a final conclusion.

These last two remarks are illustrated by the example shown in Figure 10. The data are from a

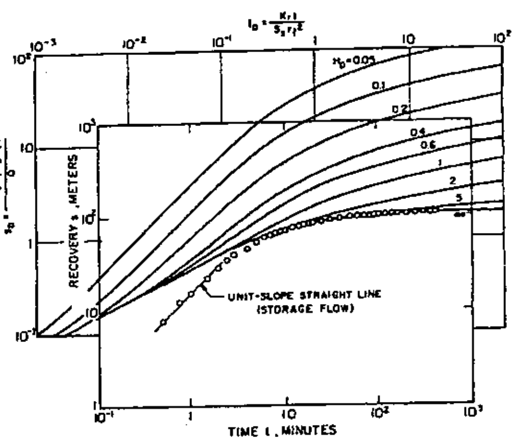

Fig. 10. Type curve matching of field data with storage flow. Recovery test, Hole UA-1-HTH-1, Amchitka Island [Ballance and Dinwiddie, 1972] t' is the time since pumping stopped.

recovery test in well UA-1-HTH-1, on Amchitka Island, following a pump test performed in the same well [Ballance and Dinwiddie, 1972]. The log-log plot on Figure 10 has an initial straight line of unit slope and the water level becomes constant at long time. The field plot can be matched against the type curve $H_D = \infty$ from Figure 8, which would suggest the presence of horizontal fractures of large volume in an aquifer of very large thickness. The drawdown behavior could also be interpreted, however as (1) a well with wellbore storage in a limited aquifer and constant head at the outer boundary, or (2) a well with wellbore storage and which only partially penetrates an aquifer of very large thickness. A knowledge of the actual aquifer conditions is thus required to decide which model should be used for analysis. For the example shown in Figure 10, the effective aquifer thickness may possibly be greater than 5,000 feet, and the well was drilled only to a depth of 3,458 feet. However, logs indicate the existence of an extensive system of nearly horizontal fractures which at some points evidently have substantial dimensions because of the significant mud losses experienced in this and adjacent boreholes nearby. Therefore, we believe that the horizontal fracture model presented in this study should be used for analysis.

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NOMENCLATURE

		Dimensions
c	= Total compressibility of aquifer (water plus rock)	L^2F^{-1}
erf(x)	= Error function of x	
Ei(-x)	= Exponential integral of x	
H	= Thickness of aquifer	L
$I_o(x)$	= Zero order modified Bessel function of the first kind	
K	= Permeability	LT^{-1}
Q	= Rate of discharge at pumping well	L^3T^{-1}
r	= Radial distance to pumping well	L
r_D	$= \frac{1}{x_f}\sqrt{x^2 + y^2 \frac{T_x}{T_y}}$ dimensionless distance to pumping well	
r_f	= Horizontal fracture radius	L
r_w	= Pumping well radius	L
s	= Drawdown	L
S	= $\gamma\phi cH$ Storage coefficient	
S_s	= $\gamma\phi C$ Specific storage	L^{-1}
t	= Time since pumping started	T
T	= KH, transmissibility	L^2T^{-1}
x,y,z	= Coordinates	L
x_f	= Vertical fracture half length	L
γ	= Specific weight of liquid	FL^{-3}
ϕ	= Porosity	

Subscripts:

D = Dimensionless

x,y,z,r = Relative to the x,y,z, or r direction.

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INTERPRETATION DES ESSAIS DE POMPAGE DANS LES AQUIFERES FISSURES

Résumé

Les courbes de rabattement obtenues lors des essais de pompage sont généralement interprétées d'aprés la solution analytique de Theis, qui n'est strictement valable que pour les aquifères homogènes, isotropes et de tres grandes dimensions.

Les courbes de rabattement dans les aquifères fissurés peuvent être trés différentes de celles prévues par Theis, en particulier si le puits de pompage communique avec une des fractures. Une telle situation peut être analysée à l'aide des solutions analytiques developpées pour l'étude des écoulements transitoires dans les réservoirs ayant subi une fracturation hydraulique, c'est à dire comportant une fracture unique, horizontale ou verticale, en communication avec le puits. L'effet des autres fractures peut être rendu en considérant l'aquifère comme anisotrope. Ainsi qu'il est indiqué sur des examples pratiques, il est possible de déceler sur les courbes de rabattement l'existence d'un système de fractures, d'en reconnaitre la nature, et de calculer les paramètres hydrauliques de l'aquifère fissuré.

EINE METHODE ZUR INTERPRETATION DER DATEM VON PUMPVERSUCHEN IN WASSERFÜHRENDEN SCHICHTEN MIT KLÜFTEN

Zusammenfassung

Die traditionelle Berechnungsmethode des nicht-stationären Verhaltens einer wasserführenden Schicht während eines Pumpversuchs beruht auf der Lösung von Theis fur Radialströmung in einem homogenen, isotropen, unendlichen Medium. Mit Hilfe der Theis'schen Lösung kann man die Permeabilität und den Speicherkoeffizienten der wasserführenden Schicht bestimmen. Die Ergebnisse werden jedoch beeinflusst durch Abweichungen von den idealisierenden Annahmen der Theis'schen Lösung.

Wenn der Brunnenschacht durch Klüfte hindurchführt, dann weicht das Verhalten der wasserführenden Schicht wesentlich von dem nach der Theis'schen Lösung vorausbestimmten Verhalten ab. Ein anderer Weg muss daher eingeschlagen werden. Analytische Lösungen für die Strömung zu einem Brunnen, der durch eine entweder horizontale oder vertikale Kluft hindurchfuhrt, sind nun vorhanden, und Standartkurven sind entwickelt worden. Mit Hilfe dieser Kurven ist es möglich, wasserführende Schichten mit horizontalen und vertikalen Klüften zu unterscheiden und das System als ein gleichwertiges anisotropes, homogenes, poröses Medium mit einer einzigen Kluft sehr viel grösserer Permeabilität zu berechnen. An Hand von Beispielen wird gezeigt, wie die neuen Standartkurven benutzt weden können, um die Art, die Ausdehnung und die Richtung einer einzelnen Kluft in dem gleichwertigen, porösen Medium zu bestimmen.