Climate, Soil, and Vegetation

6. Dynamics of the Annual Water Balance

PETER S. EAGLESON

Department of Civil Engineering, Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Mass conservation is employed to express the natural water balance of climate-soil-vegetation systems in terms of the average annual values of precipitation, evapotranspiration, surface runoff, and groundwater runoff as derived from the probability distributions of storm properties and from the physics of the appropriate storm and interstorm soil moisture fluxes. The resulting conservation equation is used to define the dimensionless parameters governing the dynamic similarity of the annual water balance. An asymptotic analysis of this water balance equation yields a set of rational criteria for the classification of climate-soil-vegetation systems. Sensitivity with respect to the primary climate, soil, and vegetal parameters demonstrates that qualitative changes in water balance behavior are primarily dependent upon the exfiltration effectiveness of the soil. A natural selection hypothesis is presented which specifies the stable vegetation density and the plant coefficient for a given climate-soil system in which water and not nutrition or light is limiting.

INTRODUCTION

Analytical formulation of the annual water balance from mass conservation principles began early in the twentieth century when hydrologists, notably Gannet [1908] and Meyer [1915], recognized that annual runoff was a residual determined by the difference between annual precipitation and annual evapotranspiration.

Thornthwaite [1944] appears to have been the first to realize that evapotranspiration must have two aspects, actual and potential, and that both play a role in the water balance depending upon the 'deficiency' or 'sufficiency' of soil moisture. He developed an empirical relationship for potential evapotranspiration as a function only of the atmospheric temperature and a 'temperature efficiency' index [Thornthwaite, 1931].

In a later classic paper, Thornthwaite [1948] used the concept of potential evapotranspiration as a factor along with precipitation in what he termed a 'rational classification' of climate. He correctly recognized the key climatic role of the precipitation-potential evapotranspiration ratio but took no account of the role of the soil properties as he performed a month by month moisture accounting to arrive at a soil moisture index of climate.

The moisture-accounting method became increasingly popular as a means of relating storm precipitation and runoff beginning perhaps with the work of Snyder [1939], who used temperature and time as an index to soil moisture. As the role of evaporation began to be understood, better, Linsley and Ackermann [1942] used cumulative pan evaporation as an index to soil moisture in their accounting scheme. Kohler and Linsley [1951] followed this with a more elaborate moisture-accounting technique, which they applied on a daily basis.

Features of these various methods are combined in the moisture-accounting method recommended today by the Soil Conservation Service [Mockus, 1964] for calculation of basin water yield on a monthly basis. Here although the so-called 'water-holding' capacity of the soil is introduced, the soil's capacity to infiltrate water and to deliver soil moisture to the surface by capillarity plays no role.

The advent of the computer made it practical to perform moisture accounting on even shorter (i.e., storm and sub-storm) time scales and to account in a more physically realistic manner for soil moisture variations. A pioneering paper in this field is the well-known work of Crawford and Linsley [1966].

In summary, we see that the trend of hydrologic water balance research has been to use the increased understanding of evapotranspiration and the power of the digital computer to elaborate schemes for evaluating the loss term in the basic conservation relation applied over small time intervals at specific locations. In so doing, however, the dynamics of the soil moisture movement processes have been represented, if at all, by gross indices which do not lend themselves to generalization. There appear to have been few attempts to use our current high level of physical understanding of all the natural processes involved to develop a generalized model of the annual water balance. Such a model might produce valuable insights into the interactive role of soil moisture in the determination of climate and would provide a tractable basis for deriving generalized probability distributions of such important water balance components as annual basin yield.

Of all previous work on this subject, that coming closest to these goals appears to be the 'evaporation climatology' formulation of Lettau [1969] and Lettau and Baradas [1973]. They write a dimensionless water balance equation in which the evaporation term is replaced by the latent heat term of an energy balance relation. In this approach, however, there is no explicit consideration of the effect of the soil and vegetal properties which will control the evaporation under all but the most humid conditions. He accomplishes this necessary shift from climate control of evaporation to soil control of evaporation (as aridity increases) in an implicit manner through the introduction of an empirical 'interpolation function' which lacks the physical basis being sought here.

WATER BALANCE EQUATION

The annual conservation of water mass for a unit watershed area is usually written

\[ PA = Rs_a + Rg_a + Er_s + AS_{sa} + AS_{sg} \]  \hspace{1cm} (1)

where

- \( PA \) annual (seasonal) precipitation, centimeters;
- \( Rs_a \) annual (seasonal) surface runoff, centimeters;
- \( Rg_a \) annual (seasonal) groundwater runoff, centimeters;
Recognizing the presence of a storm surface retention $E_r$ which is evaporated between storms [Eagleson, 1978b], we may define two new annual quantities, annual rainfall excess $R_{sa}^*$, where

$$R_{sa}^* = R_{sa} + E_r$$

and annual evapotranspiration from soil moisture $E_{sa}^*$, where

$$E_{sa}^* = E_{sa} - E_r$$

In both of these, $E_r$ equals annual surface retention. These quantities allow (1) to be rewritten to account only for that moisture $I_s$ which infiltrates the soil annually. That is,

$$I_s = P - R_{sa}^* - \Delta S_{sa} = E_{sa}^* + R_{ga} + \Delta S_{ga}$$

Taking the time average of (4) term by term and assuming the system to be stationary in the long term allow us to discard the troublesome storage terms and to write the average annual water balance

$$E[I_s] = m_{Pn} - E[R_{sa}^*] = E[E_{sa}^*] + E[E_ga]$$

in which $E[ ]$ equals the expected value of $[ ]$, and $E[P_n] = m_{Pn}$.

Using the observed probability distributions of storm properties and simplified infiltration dynamics, Eagleson [1978b] derived the average annual infiltration (for climates without significant snowfall):

$$E[I_s] = m_{Pn}[1 - e^{-G-2I(\sigma + 1)\sigma - \sigma}]$$

when

$$e^{-G-2I(\sigma + 1)\sigma - \sigma} > E[E_{sa}^*]/m_{Pn}$$

Otherwise,

$$E[I_s] = m_{Pn} - E[E_{sa}]$$

In the above,

$$G = \frac{\alpha K(1)}{2} + s_0^2 - \alpha w$$

where

$$\alpha$$ reciprocal of average rainstorm intensity, equal to $m_t^{-1}$, seconds per centimeter;

$$K(1)$$ saturated effective hydraulic conductivity of soil, centimeters per second;

$$s_0$$ space and time average soil moisture concentration in surface boundary layer;

$$w$$ apparent velocity of capillary rise from water table, centimeters per second;

$$n$$ effective porosity of soil;

$$\eta$$ reciprocal of average rainstorm depth, equal to $m_t^{-1}$, cm$^{-1}$;

$$\Psi(1)$$ saturated soil matrix potential, centimeters (suction);

$$\phi_t$$ dimensionless infiltration sorptivity;

$$\delta$$ reciprocal of average rainstorm duration, equal to $m_t^{-1}$, s$^{-1}$;

$$c$$ soil pore disconnectedness index;

$$m$$ soil pore size distribution index.

Eagleson [1978b, c] also gives, to the first approximation,

$$E[E_{g}]/\bar{e}_p = m_{e}E[E_{g}]/\bar{e}_p = m_{e}(1 - M)\left[1 - e^{-\delta h_0/\bar{e}_p}\frac{I[x, \lambda M]}{\Gamma(\kappa)}\right]$$

$$+ m_{k_{s}M}\left[1 - e^{-\delta h_0/\bar{e}_p}\frac{I[x, \lambda k_{s}h_0]}{\Gamma(\kappa)}\right]$$

$$- \left[1 + \frac{\beta h_0/\bar{e}_p}{\lambda h_0}\right]^{-\kappa} \frac{I[x, (\lambda h_0 + \beta h_0/\bar{e}_p)]}{\Gamma(\kappa)}$$

where

$$m_e$$ mean number of storms per rainy season;

$$h_0$$ storm surface retention capacity, centimeters;

$$k_s$$ plant coefficient (equal to potential rate of transpiration divided by potential (soil surface) rate of evaporation $\bar{e}_p/\bar{e}_p$ and approximately equal to the effective transpiring leaf surface per unit of vegetated land surface);

$$M$$ vegetated fraction of surface;

$$\kappa$$ parameter of gamma distribution of storm depth;

$$\lambda = \kappa/m_{ht}$$. cm$^{-1}$

from which he finds the average annual surface runoff to be

$$E[R_{sa}] = m_{Pn}e^{-G-2I(\sigma + 1)\sigma - \sigma} - E[E_{sa}]$$

when

$$e^{-G-2I(\sigma + 1)\sigma - \sigma} > E[E_{sa}]/m_{Pn}$$

Otherwise,

$$E[R_{sa}] = 0$$

Using the time average rate of potential evapotranspiration, the probability distribution of interstorm periods, and simplified exfiltration dynamics, Eagleson [1978c] derived the annual (rainy seasonal) average total evapotranspiration $E[E_{sa}]$ as

$$E[E_{sa}] = E[E_{sa}]J(E, M, k_s, h_0)$$

where $J(E, M, k_s, h_0)$ is the evapotranspiration function.

$E[E_{sa}]$ is the weighted average rainy season potential evapotranspiration and is given by [Eagleson, 1978c]

$$E[E_{sa}] = m_{Pn}e^{-G-2I(\sigma + 1)\sigma - \sigma} - E[E_{sa}]$$

We can find the average annual evapotranspiration from soil moisture $E[E_{sa}]$ from (3) as

$$E[E_{sa}^*] = E[E_{sa}] - E[E_{sa}]$$

To within 2% or 3% error, we can satisfy (13) by setting the storm surface retention capacity $h_0$ equal to zero in the expression for $J(E, M, k_s, h_0)$, while replacing the potential total evapotranspiration $E_{ea}$ by the potential soil moisture evapotranspiration $E_{sa}^*$. That is,

$$E[E_{sa}^*] = E[E_{sa}^*]J(E, M, k_s)$$
in which, accounting for the reduced opportunity for soil moisture evapotranspiration,

\[ E[E_{p_a}^*] = E[E_{p_a}] - E[E_{p_a}] \]  

and

\[ J(E, M, k_0) = 1 - \left[ \frac{1 - M}{1 - M + M k_0} \right] \cdot \left( [(1 + M k_0 + (2B)^{1/2}) e^{\sigma E} - (M k_0 + (2C)^{1/2}) e^{c E} - (2E)^{1/2} (\gamma(1, CE) - \gamma(1, BE))] \right) \]  

where

\[ B = \frac{1 - M}{1 + M k_0 - w/\bar{\varepsilon}_p} + \frac{M k_0 + (1 - M)w/\bar{\varepsilon}_p}{2(1 + M k_0 - w/\bar{\varepsilon}_p)^2} \]  

\[ C = \frac{\gamma(M k_0 - w/\bar{\varepsilon}_p)^2}{(1)} \]  

where \( w \) is the apparent velocity of capillary rise from the water table and

\[ E = \frac{2dnk(1)\Psi(1)\phi_s - \sigma^{2}}{\sigma m e^2} \]  

where \( \beta \), the reciprocal of the average time between storms, equals \( m \) in \( s^{-1} \), \( \phi_s \) is the dimensionless desorption diffusivity, and \( d \) is the diffusivity index of the soil.

The average annual groundwater runoff is given by the difference between the average wet season percolation to the water table and the average annual capillary rise from the water table. By using the components derived by Eagleson [1978d] this difference becomes

\[ E[R_a] = m_k k(1) s_k - Tw \]  

where \( m_k \) is the average length of the rainy season in seconds and \( T \) is 1 year in seconds; and where the velocity of capillary rise \( w \) is given by [Eagleson, 1978d]

\[ w = (1 + \frac{\Psi(1)}{Z})^{-1} \]  

\[ Z \] being the depth to the water table in centimeters.

Substituting (6), (14), and (20) in (5) gives the average annual water balance equation for soil moisture,

\[ m_{p_a}[1 - e^{-\sigma E}] (\gamma + 1) e^{-\sigma E} = E[E_{p_a}^*]J(E, M, k_0) + m_k k(1) s_k - Tw \]  

When the surface runoff is nonzero. (The term to the left of the equal sign is infiltration, the first term to the right is evapotranspiration from soil moisture, and the last two terms are groundwater runoff (the first is groundwater recharge and the last is groundwater loss).) Otherwise,

\[ m_{p_a} = E[E_{p_a}] J(E, M, k_0, h_0) + m_k k(1) s_k - Tw \]  

**Sensitivity**

To understand more fully the physical significance of (22), it is helpful to examine the sensitivity of the water balance element to changes in the soil, climate, and vegetal parameters. The highlights of such an examination are presented in this section on the basis of (22a), the more widely applicable of the two relations. A more detailed analysis is given elsewhere [Eagleson, 1978f].

We will begin by examining, in Figure 1, the effect of the primary soil properties \( k(1) \) and \( c \) on the average annual water balance components in two very different climatic regimes under the simplifying restrictions of no vegetation \( (M = 0) \) and no surface retention \( (h_0 = 0) \). It will be seen later that the bare soil (i.e., \( M = 0 \)) behavior characterizes the system evapotranspiration quite well (qualitatively); thus this simplification will not distort the sensitivity analysis. Neglect of the surface retention will produce significant distortion only in the most arid climates.

The climatic parameters for the two locations studied, Clinton, Massachusetts, and Santa Paula, California, are given in Table 1. Because the effective porosity has a very limited range (in comparison with \( k(1) \) and \( c \), we will keep it constant throughout this study at the value \( n = 0.35 \).

On Figure 1, we should note the following.

1. Even though we are discussing the behavior of mean annual values, we have, for convenience, dropped the 'mean' and 'expectation' notations.

2. The number written following the label on the dependent variable axis is the maximum plotted value of that variable.

3. The indicated classification of the systems as subhumid or arid will be explained later.

To understand the physical significance of these figures, we need to remember that as \( \log k(1) \) increases, \( k(1) \) increases, and the soil becomes intrinsically more permeable. The parameter \( c \) measures the disconnectedness of the soil pores; thus smaller values will indicate higher permeability (for constant \( k(1) \) and \( s_k \)).

**Soil moisture.** Look first at the \( s_k - k(1) \) plane in Figure 1a, which represents the subhumid climate of Clinton, Massachusetts. Keeping \( c \) constant at its origin value of 4 and increasing the intrinsic permeability \( k(1) \), we find (although it is not shown) that for very small \( k(1) \) the soil moisture increases with \( k(1) \) due to the increasing ability of the soil to accept moisture. As \( k(1) \) continues to increase, beyond the origin value of \( 10^{-11} \) cm², the soil becomes unable to hold the infiltrated water against gravitational percolation, and the soil moisture falls off. This is accompanied by a rapid rise in the percolation to groundwater as can be seen at the bottom of this figure. Holding \( c \) constant at the large value, \( c = 11 \), we find a relative insensitivity of \( s_k \) over the full range of \( k(1) \) because of the extremely small permeabilities produced there by the factor \( s_k \).

In Figure 1b, which corresponds to the Santa Paula, California, climate, we see much less sensitivity of \( s_k \) to either soil property and a shift of the maximum soil moisture to the region of low \( k(1) \) and high \( c \), a region of low permeability. To explain the qualitative difference between Figures 1a and 1b, we need to examine the fundamental difference between the two climates as demonstrated by the variability of their annual evapotranspiration. This is shown in the second row of Figure 1.

**Evapotranspiration.** In Clinton, the rate of potential evaporation is about one half that for Santa Paula, while \( \beta \) for Clinton is about 3 times that of Santa Paula. These two factors combine to make the exfiltration effectiveness \( 2E(1) \) (twice (19) with \( s_k = 1 \)) large for Clinton and small for Santa Paula. This means that evapotranspiration in Clinton is limited by the climatically imposed potential value, while that in Santa Paula will be controlled by the soil.

Therefore in Clinton, where there is inadequate atmospheric moisture capacity, we expect the actual evapotranspiration
Fig. 1. Sensitivity of annual water balance to changes in soil parameters ($M = 0, \delta_a = 0$ and $w/\delta_a << 1$).
TABLE 1. Independent Soil and Climate Parameters for Two Climate-Soil Systems

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Clinton, Mass.</th>
<th>Santa Paula, Calif.</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>k(1), cm/s²</td>
<td>2.8 x 10⁻⁶</td>
<td>1.2 x 10⁻⁹</td>
</tr>
<tr>
<td>c</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>m_0, cm</td>
<td>111.3</td>
<td>54.4</td>
</tr>
<tr>
<td>e_0, cm/d</td>
<td>0.15</td>
<td>0.27</td>
</tr>
<tr>
<td>m_a, d</td>
<td>3.0</td>
<td>10.4</td>
</tr>
<tr>
<td>m_v, d</td>
<td>0.32</td>
<td>1.4</td>
</tr>
<tr>
<td>z_v</td>
<td>362</td>
<td>212</td>
</tr>
<tr>
<td>T_a (°C)</td>
<td>8.4</td>
<td>13.8</td>
</tr>
<tr>
<td>h_a, cm</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>k_v</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

 rate to be insensitive to the soil’s ability to deliver moisture to the surface and hence insensitive to changes in the soil properties. Looking at Figure 1a, we see that in Clinton, for low to moderate c, the evapotranspiration E_v/P_a is insensitive to k(1) and to c, being limited by and essentially equal to the potential value. As the soil becomes very impermeable (high c and low k(1)), E(1) will decrease, ultimately putting the evapotranspiration under soil control and causing a decline in E_v/P_a.

In Santa Paula, where there is excess atmospheric moisture capacity, we expect the evapotranspiration to follow the soil’s ability to deliver moisture to the surface and hence to be sensitive to changes in soil properties. This is borne out in Figure 1b.

Thus where the evapotranspiration is controlled by the climate’s evaporation potential rather than by the moisture supply to the surface, the soil moisture will be highest in the region where the permeability readily admits water and holds it effectively against gravity. This is the low c moderate k(1) region for a humid climate.

Where the evapotranspiration is controlled by the moisture supply to the surface, either by the annual precipitation itself or by the soil properties, the lowest soil moistures will occur where the soil properties most effectively permit soil moisture movement. This will occur in the region of largest E(1), which is small c and large k(1). Conversely, because of difficulty of moisture movement the largest soil moisture will occur where k(1) is small and c is large.

Surface runoff. The behavior of surface runoff is qualitatively the same in both climates. The surface runoff component R_s/P_a increases with increasing c due to decreasing permeability and decreases with increasing k(1) due to increasing permeability.

Groundwater runoff. The effect of soil characteristics on groundwater runoff is shown in the last row of Figure 1. For a given c, R_g/P_a increases monotonically with k(1), the rate being governed by the particular value of c and by the soil moisture s_v. Since the Santa Paula soil moisture is smaller than that for Clinton, the rates of increase are larger in the latter case. For large values of k(1) we note a saddle in R_g/P_a as c increases, particularly for Santa Paula. This results from the behavior of the factor s_v⁻², where s_v is less than one and increases with c.

Climate parameters. A similar study of sensitivity to changes in the climate parameters [Eagleson, 1978] merely reinforces the above observation that qualitative changes in behavior of climate-soil systems are linked directly to the degree of humidity or aridity as defined by the primary parameter E. Any soil or climate change which significantly alters E will cause a qualitative alteration in the water budget. In approximate order of importance in their effect upon E are (1) the rate of potential evaporation E_p, (2) the annual precipitation P_a (through s_v), (3) the season length r, and (4) the average time between storms m_v.

This dominance of E is illustrated most strikingly by another sensitivity illustration. In Figure 2 we compare the water balance components of the Santa Paula climate-soil system (Figure 1b) with those of the Clinton climate-soil system, in which the rate of potential evaporation has been raised to the Santa Paula value. In both cases we have again used the M = 0 and h_0 = 0 simplification.

Notice in Figure 2 that changing only the rate of potential evaporation causes the Clinton water balance surfaces to take on soil-controlled shapes identical to, but slightly displaced from, the corresponding Santa Paula surfaces. The displacement results primarily from the different season length which causes higher E_p/P_a in Clinton and hence smaller runoff components there.

Vegetation. To understand the system sensitivity to vegetal characteristics, we must first recall and restate the principal assumptions utilized in modeling the interaction between vegetation and soil moisture [Eagleson, 1978d].

We speak of an ‘equilibrium’ state of our climate-soil-vegetation system. By this, we do not mean a state in which all evolution has ceased but rather one in which the natural evolutionary change over ‘engineering time’ is small enough to be negligible at the desired level of accuracy.

More importantly perhaps, we are considering only those vegetal systems which are water limited as opposed to those that are limited by nutrient supply, by available light, or by other ecosystem factors.

We have hypothesized that under natural equilibrium conditions, each vegetal species will operate, on the average, in the unstressed state. This is proposition 1 of our natural selection hypothesis for equilibrium of natural vegetal systems. This hypothesis permitted us to approximate the average annual rate of transpiration per unit of vegetated surface by the potential value k_w E_p.

We next assumed that the canopy density and root system have evolved so that at natural equilibrium they draw soil moisture uniformly from the entire volume above the maximum root depth, i.e., from the soil beneath the unvegetated surface fraction as well as from that beneath the vegetated portion. This is proposition 2 of our natural selection hypothesis. That is, natural vegetal systems exist at an equilibrium density and species mix which makes full use of the available soil moisture in the root zone.

These two assumptions lead us to one-dimensional modeling of the root extraction of soil moisture as a distributed sink of aggregate strength M_k_w E_p, which acts in a way that reduces the bare soil exfiltration rate. In the presence of adequate nutrients and light this rate of water use should be proportional to the rate of biomass production by the vegetal cover.

With this background, we will now examine the role of vegetal canopy density in the average annual water balance. For a given set of soil and climate parameters and for a given k_w, (22) does not define s_v uniquely but rather as a function of M. To explore the form of this function, we have used the four
representative sets of soil parameters listed in Table 2. The $S_o$ versus $M$ relations, obtained from (22a) by using all four soils, are presented in Figure 3 for both the Clinton and Santa Paula climates under conditions of average annual precipitation, i.e., $P_A/m_p = 1$ and $k_o = 1$. Notice that there is a particular $M = M_o$ for each climate-soil (and $k_o$) combination at which $S_o$ is a maximum. This may be explained as follows. For small but nonzero $M$, where bare soil exfiltration is the dominant mode of soil moisture depletion, the composite evapotranspiration rate will be reduced over that for bare soil yielding higher equilibrium soil moisture. As $M$ increases, the vegetated fraction, transpiring at the potential rate, becomes dominant and the composite evapotranspiration rate will begin to increase, thereby reducing the equilibrium soil moisture. This peak in the $S_o$ versus $M$ curve is apparent at Clinton only for the less permeable clay and clay loam soils because it is only for these soils that the evapotranspiration is not controlled by the climate (see Figure 1). The point of maximum $S_o$ in Figure 3 corresponds to maximum surface and groundwater runoff, which means for fixed precipitation there is minimum evapotranspiration from soil moisture. We thus expect a minimum in $E[E_r,*] = E[E_p,*](E, M, k_o)$ at $M = M_o$. For the common
case, \( k_o = 1 \), all \( M \) sensitivity comes from \( J(E, M, k_o) \), which will then have a minimum at \( M = M_0 \). We see this in Figure 4, where the information of Figure 3 is used (with (16)) to display the corresponding \( J \) versus \( M \) relations. Note that in Santa Paula, the clay and clay loam soils cannot absorb enough water to produce canopy densities greater than 0.4 and 0.8, respectively, as long as \( k_o = 1 \).

For a given \( M \), (22) does not define \( S_o \) uniquely but rather as a function of \( k_o \). In Figure 5 we see that for constant \( M \) and \( P_A/m_{PA} \) (0.5 and 1, respectively, in this case), increasing the plant coefficient \( k_o \) causes a monotonic decline in \( S_o \). This decline becomes precipitous at a value of \( k_o \) which is very sensitive to climate for a given soil, being smaller for arid climates.

We thus see that there is an infinity of \( M-k_o \) combinations which can satisfy (22) for a given climate and soil.

**Vegetal Equilibrium Hypothesis**

While \( M \) may be evaluated observationally and \( k_o \) may be estimated from the literature once the predominant species are known, it is natural to wonder what determines the \( M-k_o \) values in a given climate-soil system.

It seems reasonable to assume that in natural systems the plant coefficient changes on an evolutionary time scale while the canopy density may change in response to short-term variability in climate. We thus hypothesize separate equilibrium states: (1) a short-term or growth equilibrium reached by the canopy density of a given species (i.e., of given \( k_o \)) in response to short-term fluctuations of soil moisture and (2) a long-term or evolutionary equilibrium reached by natural selection of species to be optimally compatible with the given climate and soil. We seek first to propose principles that guide the selection of the equilibrium canopy density of a given species.

We hypothesize that natural vegetal systems of given species will develop a canopy density which produces minimum stress under the local climatic conditions. A necessary condition for zero stress is a soil moisture which is sufficiently larger than the largest critical soil moisture in a given species mix, so that stress will not be produced during the periods of dry weather. A necessary condition for minimum stress is that the soil moisture take on the maximum value possible. This is proposition 3 of our natural selection hypothesis. That is, natural vegetal systems of given species tend toward a growth equilibrium in which soil moisture is maximized.

If we use this hypothesis, the given climate, soil, and plant coefficient determine the equilibrium canopy density \( M = M_o \) through the water balance equation, where the soil moisture is maximum or, equivalently, where the soil moisture evapotranspiration is a minimum (see Figures 3 and 4). Differentiating, we have the new relationship

\[
\frac{\partial E[E_{T,*}]}{\partial M} = J(E, M, k_o) \frac{\partial E[E_{PA,*}]}{\partial M} + E[E_{PA,*}] \frac{\partial J(E, M, k_o)}{\partial M}
\]

(23)

in which \( M = M_o \) and \( J(E, M, k_o) \) are given by (16). Exactly for \( k_o = 1 \) and within about 2% otherwise, (23) can be replaced by

\[
\frac{\partial J(E, M, k_o)}{\partial M} = 0 \quad M = M_o
\]

(24)

Substitution of (24) in (16) and letting \( M = M_o \) give the equilibrium soil moisture evapotranspiration function \( J(E, M_o, k_o) \) for natural climate-soil-vegetation systems. This is plotted as the solid lines in Figure 6 for values of \( k_o \) covering the
Fig. 5. Sensitivity of mean annual soil moisture to plant coefficient for typical soils (M = 0.5 and w/do << 1).

We come now to the plant coefficient b. The primary appearance of b in all cases is through the rate of potential evapotranspiration from vegetated surfaces Mdo = Mv/kv. We have already pointed out that while the range of this parameter is very important for a first approximation, to the first approximation M will remain constant at M0 = 0.5.

Also shown as dashed lines on Figures 6 and 7 are the bare soil (M0 = 0) evaporation function and its asymptotes. It is very important for a later argument to note that the asymptotes of J(E, M0, kv) are identical with those of J(E).

We come now to the plant coefficient b. The primary appearance of b is through the rate of potential transpiration from vegetated surfaces Mdo = Mv/kv. We have already pointed out that while the range of this parameter is very important for a first approximation, to the first approximation M will remain constant at M0 = 0.5.

In the upper portions of Figure 8 we present the variation of S with changes in both M and kv. We have already pointed out that while the range of this parameter is very important for a first approximation, to the first approximation M will remain constant at M0 = 0.5.

In the lower portions of Figure 8 the set of values (M0 and kv) defining the locus of S maxima in each case is plotted to show the variability of Mv along the locus. Along M = 1, S has only a local maximum, since for small kv, the derivative (24) vanishes at the infeasible density M > 1.

Fig. 6. Evapotranspiration function for natural systems (w/do << 1).

The values of M0 and kv are determined as they are above for each climatic combination and are listed in Table 3. Notice that both M0 and kv get smaller for a given soil as the climate becomes more arid. Since many (perhaps most) natural systems will be nutrient limited or light limited and hence will operate at a lower value of kv, these values are not limiting and the average rate of water use Mv,kv will be proportional to the average rate of production of vegetal biomass. It has been hypothesized that natural systems evolve toward a condition of maximum biomass production. These propositions lead to propositions of our natural selection hypothesis.

In the upper portion of Figure 8 we present the variation of S through its surrogate 2.1 - E = E, with changes in both M and kv. In the lower portion of Figure 8 the set of values (M0 and kv) defining the locus of S maxima in each case is plotted to show the variability of Mv along the locus. Along M = 1, S has only a local maximum, since for small kv, the derivative (24) vanishes at the infeasible density M > 1.
the index of potential biomass and will retain the suboptimal notation for \( k_v \).

Many questions remain concerning the validity of this hypothesis which must be answered through comparison with field observations. In a later paper [Eagleson, 1978e] we will compare these hypothesized equilibrium states with those for which our derived annual yield frequency is in best agreement with the observed annual streamflow frequency at Clinton and at Santa Paula.

**ASYMPTOTIC BEHAVIOR**

Equation (22a) defines the soil moisture balance in terms of 23 variables and parameters, \( m_{r}, m_{s}, m_{e}, m_{c}, m_{n}, m_{H}, m_{m}, m_{r}, c, d, m, n, \Psi(1), \phi_{r}, \phi_{v}, Z, s_{o}, k(1), k_{o}, M_{o}, k_{o}, \) and the normal annual surface temperature \( T_{a} \). We have, in addition to (22a), nine supplementary relations:

By definition

\[
m_{H} = m_{P_{s}}/m_{r} \tag{26}
\]

By definition

\[
m_{r} = m_{o}(m_{s} + m_{r}) \quad m_{s} \geq 1 \tag{27}
\]

With the assumption of independence of \( i \) and \( t_{r} \)

\[
m_{i} = m_{H}/m_{r} \tag{28}
\]

From Brooks and Corey, [1966]

\[
m = 2/(c - 3) \tag{29}
\]

By definition

\[
d = (c + 1)/2 \tag{30}
\]

From Eagleson [1978d] (empirical)

\[
\Psi(1) = \Psi(n, k(1), T_{a}) \tag{31}
\]

From Eagleson [1978d]

\[
\phi_{i} = \phi_{i}(d, s_{o}) \tag{32}
\]

From Eagleson [1978d]

\[
\phi_{s} = \phi_{s}(d) \tag{33}
\]

From natural selection hypothesis (24)

\[
M_{o} = M(s_{o}) \tag{34}
\]

To solve the water budget relation for the dependent variable \( s_{o} \), we must therefore specify the values of 13 parameters (i.e., 23 variables minus 10 equations).

For the soil system we have the six independent parameters \( h_{0}, k(1), c, n, T_{a}, \) and \( Z \).

For the vegetation, \( k_{o} \) is the only independent parameter.

For the climate we may choose six independent parameters from among the set of seven, \( \kappa, m_{P_{s}}, \epsilon_{p}, m_{H}, m_{s}, m_{r}, \) and \( m_{r} \).

Let us now examine the qualitative behavior of (22a) by allowing the individual mean annual water balance components to vary with the mean annual precipitation, while keeping all other independent variables constant.

Consider the simple case in which the water table is at \( Z = \infty \). Since we are varying \( m_{P_{s}} \), (26) tells us that either \( m_{H} \) or \( m_{r} \) (or both) must vary also. For simplicity of argument we will hold \( m_{H} \) fixed as an independent variable over the full range of \( m_{P_{s}} \) and accomplish the \( m_{P_{s}} \) variation solely through variation in \( m_{r} \).

As \( m_{P_{s}} \to 0 \),

\[ s_{o} \to 0 \tag{35} \]

and

\[ G \to G(0) = G(1)/2 = \alpha K(1)/2 \tag{36} \]

The separate terms of (22a) will approach the asymptotes given by

\[ E[R_{s}] = m_{H}K(1)s_{o} \to 0 \tag{37} \]

\[ E[I_{s}] = m_{P_{s}}[1 - e^{-G(0)}] = m_{P_{s}}(1 - 1 + 1) = m_{P_{s}} \tag{38} \]

We saw earlier (Figure 6) that the asymptotes of \( J(E, M_{o}, k_{o}) \) are identical with those for the bare soil evaporation function \( J(E) \). These have been shown to be [Eagleson, 1978c]

\[ J(E, M_{o}, k_{o}) \to J(E) \to 1 \quad s_{o} \to 1 \tag{39} \]

and

\[ J(E, M_{o}, k_{o}) \to J(E) \to \frac{[\pi E/2]^{1/2}}{0} \tag{40} \]
Fig. 8. Maximization of vegetal biomass production \( (P_a/m_{P_a} = 1) \).
TABLE 3. Equilibrium Properties of Vegetal Cover

<table>
<thead>
<tr>
<th></th>
<th>Clay</th>
<th>Clay Loam</th>
<th>Silty Loam</th>
<th>Sandy Loam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_o$</td>
<td>$k_o$</td>
<td>$M_o$</td>
<td>$k_o$</td>
</tr>
<tr>
<td>Clinton, Mass.</td>
<td>0.2</td>
<td>2.1</td>
<td>0.90</td>
<td>0.70</td>
</tr>
<tr>
<td>Santa Paula, Calif.</td>
<td>0.1</td>
<td>0.7</td>
<td>0.75</td>
<td>0.30</td>
</tr>
</tbody>
</table>

We thus have

$$E[E_{p_a}^*] \rightarrow E[E_{p_a}^*][\pi E/2]^{1/2} \quad m_{p_a} \rightarrow 0$$  (41)

where [Eagleson, 1978c]

$$E[E_{p_a}^*][\pi E/2]^{1/2} \sim s_o^{1+(c+1)/4}$$

Since $c > 3$ [Eagleson, 1978d], $c > 1 + (c + 1)/4$, and for $s_o << 1$,

$$s_o^c << s_o^{1+(c+1)/4}$$

and the groundwater recharge term will become negligible very rapidly as $s_o \rightarrow 0$. Equation (5) then gives

$$E[I_a] \rightarrow E[E_{p_a}^*] \quad m_{p_a} \rightarrow 0$$  (42)

which implies that the $m_{p_a} \rightarrow 0$ asymptotes of these two terms are identical. Equating them, we have, as $m_{p_a} \rightarrow 0$,

$$E[E_{p_a}^*][\pi E/2]^{1/2} = \left[1 - e^{-G(0)-2\sigma(0)(\sigma(0) + 1)\sigma(0)-\sigma(0)}\right]^{-1}m_{p_a}$$  (43)

For large $m_{p_a}$, the evapotranspiration term approaches the asymptote

$$E[E_{p_a}^*] \rightarrow E[E_{p_a}^*] \quad m_{p_a} \rightarrow \infty$$  (44)

The intersection of the evapotranspiration asymptotes (41) and (44) occurs at $E = 2\pi$, which separates soil-controlled from climate-controlled evapotranspiration [Eagleson, 1978c]. Putting $E = 2\pi$ in (43) gives

$$m_{p_a}/E[E_{p_a}^*] = \left[1 - e^{-G(0)-2\sigma(0)(\sigma(0) + 1)\sigma(0)-\sigma(0)}\right]^{-1}$$

and is larger for rainfall excess producing soils such as clays than it is for sandy soils which have a high infiltration capacity. For very sandy soils the rainfall excess vanishes asymptotically. Using (22b) now gives

$$m_{p_a} = E[E_{p_a}^*] \sim E[E_{p_a}^*]$$  (46)

which corresponds to the (common) $s_o \rightarrow 0$ asymptote of $E[I_a]$ and of $E[E_{p_a}^*]$, which has its steepest possible slope, $45^\circ$. To justify the above approximation, we combine (12), (15), and (27) to obtain

$$E[E_{p_a}^*] = \frac{m_0 \epsilon_p}{(1 + m_0/m_0)} \left\{1 - \frac{E[E_{p_a}^*]}{m_0 \epsilon_p}\right\}$$  (47)

Remember that we are keeping $m_0$ constant, which means that as $m_{p_a} \rightarrow 0$, $m_0 \rightarrow 0$ and $m_0 \rightarrow m_0$ and gives

$$m_0/m_0 << 1$$

At this end of the climate spectrum we may thus consider $E[E_{p_a}^*]$ to be independent of $m_{p_a}$ and approximated by

$$E[E_{p_a}^*] \rightarrow m_0 \epsilon_p - m_0 E[E_{p_a}^*] = 0$$

At the other extreme we recognize that $m_{p_a}$ cannot approach infinity solely through increase of $m_0$ while we maintain constant $m_p$, $m_u$, and $m_{p_a}$ because the product $m_0 m_u$ cannot exceed $m_r$. Therefore in what follows, we tacitly assume the limiting soil moisture to be reached at a large $m_{p_a}$ before this $m_r$ limit becomes effective.

As $m_{p_a} \rightarrow \infty$,

$$E[I_a] \rightarrow E[I_a] \quad I_a \rightarrow 0$$

$$J(E, M_o, k_o) \rightarrow J(E) \rightarrow 1$$

and

$$G \rightarrow G(1) = (m_0/m_0)K(1) = \text{const}$$

The mean annual potential soil moisture evapotranspiration is still governed in this limit by (47), which now says since $m_0 \rightarrow 0$,

$$E[E_{p_a}^*] \rightarrow 0 \quad m_{p_a} \rightarrow \infty$$

It should be recognized, of course, that in this climatic extreme, cloud cover and low temperature would also operate to decrease $E[E_{p_a}^*]$ through their important effect upon the rate of potential evapotranspiration $\epsilon_p$. Since the right-hand side of (22a) approaches a constant in this limit, the left-hand side will approach the same value. This horizontal asymptote is given, for $Z = \infty$ (and hence $w = 0$), by

$$E[I_a] = m_0 K(1) \quad m_{p_a} \rightarrow \infty$$  (48)

Equation (48) implies that as $m_{p_a} \rightarrow \infty$, the mean annual rainfall excess and groundwater runoff approach the respective asymptotes

$$E[R_{s_a}^*] = m_{p_a} - E[I_a] = m_{p_a} - m_0 K(1)$$  (49)

and

$$E[R_{s_a}] = m_p K(1)$$  (50)

With these asymptotic behaviors in mind, we present the mean annual water balance components as a function of mean
The mean annual precipitation, as is shown in Figure 9. The climate classification shown on this drawing is explained in the next section of this paper.

In the above analysis we have assumed no 'downstream control' on the lateral movement of the groundwater component of runoff. Rather, we have prescribed a fixed water table elevation independent of the groundwater discharge and of the aquifer transmissivity. This has allowed the soil to approach saturation asymptotically with increasing \( m_{pa} \) and has kept the water table from rising to the surface.

A more practical water balance situation arises from the presence of limitations on the lateral transmissivity of the soil. This situation is sketched in Figure 10. We see that whenever the saturated percolation to groundwater exceeds the limiting lateral groundwater flow \( R_{d_{max}} \), i.e., for

\[
R_{d_{max}} < m_{k}(1) \tag{51}
\]

the surface will be saturated continuously and give

\[
E[E_{\alpha_{s}}] = E[E_{\alpha_{s}}] \tag{52}
\]

and the infiltration becomes constant at the maximum value

\[
E[I_{a}] = R_{d_{max}} + E[E_{\alpha_{s}}] \tag{53}
\]

**HYDROLOGIC CLASSIFICATION OF CLIMATE-SOIL-VEGETATION SYSTEMS**

The mean annual precipitation at which (52) and (53) apply defines the condition for the existence of a swamp. From (22) evaluated at saturation, we have

\[
m_{pa} \geq [E[E_{\alpha_{s}}] + R_{d_{max}}]/[1 - e^{-Q(1)}] \tag{54}
\]

This region is indicated in Figure 10 beginning at the point 4.

Another extreme condition can be recognized for very low values of \( m_{pa} \). There is a minimum value of the soil moisture, for each vegetal species, below which that species is unable to extract moisture from the soil. Slatyer [1967, p. 76] sets this value at an average soil matrix potential of 15 bars (suction) for a variety of plants. This gives the limiting soil moisture as

\[
xo = [-\Psi(1)/15]^{m} \tag{55}
\]

which will be very small. Equation (55) is used along with the soil moisture balance equation (22) to define the value of \( m_{pa} \), below which the system may be classified as a vegetal desert.

The intersections with \( E[E_{\alpha_{s}}] \) of \( m_{pa} \) and of the dry asymptote of \( E[E_{\alpha_{s}}] \) and the intersection of the two asymptotes of \( E[I_{a}] \) separate differing regimes of hydrologic behavior due to the dominance of different terms of the soil moisture conservation equation, and they can therefore be used to define a rational classification for hydrologic climate (during the wet season) by using the classical terminology of Thornthwaite [1948]. These intersections are indicated by vertical dashed lines 1, 2, and 3 in Figures 9, 10, and 11, respectively.

The most obvious of these climatic regimes is that defined by the ratio \( m_{pa}/E[E_{\alpha_{s}}] \), which measures the potential humidity of a climate. Potentially humid climates will have

\[
m_{pa}/E[E_{\alpha_{s}}] \geq 1 \tag{56}
\]

The climate is termed 'potentially' humid because realization of the humidity depends upon the ability of the soil to infiltrate moisture and then return it to the surface for evaporation.

The soil system on the other hand is free to behave humidly whenever both interfacial moisture exchange processes, \( E[I_{a}] \) and \( E[E_{\alpha_{s}}] \), are controlled not by the soil properties but by the climate. This condition is indicated whenever the functions \( E[I_{a}] \) and \( E[E_{\alpha_{s}}] \) are dominated by their \( m_{pa} \), \( \rightarrow \infty \) asymptotes.

A necessary condition for humid behavior is thus obtained from the intersection of the \( E[E_{\alpha_{s}}] \) asymptotes as provided in (45) and as indicated by the points marked 2 in Figures 9, 10, and 11. This condition is

\[
m_{pa}/E[E_{\alpha_{s}}] \geq [1 - e^{-Q(0)-2e(0)}][\sigma(0 + 1)\sigma(0) - \sigma(0)]^{-1} \tag{57}
\]

A second necessary condition for humid behavior is obtained from the intersection of the \( E[I_{a}] \) asymptotes. Because \( E[E_{\alpha_{s}}] \) and \( E[I_{a}] \) share a common dry (i.e., soil-controlled) asymptote, we can obtain this intersection from (45) by linear proportion. Normally, \( m_{k}(1) >> E[E_{\alpha_{s}}] \); thus regardless of lateral transmissivity, we may approximate this intersection by

\[
m_{pa}/m_{k}(1) = [1 - e^{-Q(0) - 2e(0)}][\sigma(0 + 1)\sigma(0) - \sigma(0)]^{-1} \tag{58}
\]

which establishes the point marked 3 in Figures 9, 10, and 11.

In this model, soil moisture evapotranspiration is an abstraction from infiltration; thus

\[
m_{k}(1) > E[E_{\alpha_{s}}] \tag{59}
\]

which means that \( m_{pa} \) as given by (58) will always exceed \( m_{pa} \) as given by (45), and thus the necessary and sufficient condition for fully humid behavior is

\[
m_{pa}/m_{k}(1) \geq [1 - e^{-Q(0) - 2e(0)}][\sigma(0 + 1)\sigma(0) - \sigma(0)]^{-1} \tag{60}
\]
TABLE 4. Illustrative Classification of Two Climate-Soil-Vegetal Systems

<table>
<thead>
<tr>
<th>Classification</th>
<th>Equation Defining Classification Boundary</th>
<th>Value of ( m_{PA} ) at Classification Boundary*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arid</td>
<td>(46)</td>
<td>Clinton, Mass.†</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Santa Paula, Calif.‡</td>
</tr>
<tr>
<td></td>
<td></td>
<td>47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>631</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1830</td>
</tr>
</tbody>
</table>

*Values are given in centimeters.
†For Clinton, Massachusetts, \( m_{PA} = 111 \) cm.
‡For Santa Paula, California, \( m_{SA} = 54 \) cm.

There can be no climate control of soil moisture processes for \( m_{PA} \leq E[\gamma_{PA}^*] \); thus the necessary and sufficient condition for fully arid behavior is

\[ m_{PA}/E[\gamma_{PA}^*] < 1 \] (61)

The point marked 1 in Figures 9, 10, and 11 is located at the boundary of this arid zone where \( m_{PA} = E[\gamma_{PA}^*] \).

Equation (45) locates the point marked 2 in Figures 9, 10, and 11 and serves to divide the partially arid, partially humid region into two additional climatic types.

For \( m_{PA} \) larger than \( E[\gamma_{PA}^*] \) but smaller than (45), the climate is potentially humid, but the infiltration and evapotranspiration are under soil control. We will call this region 'semiarid.'

For \( m_{PA} \) larger than that in (45) but smaller than that in (58) the climate is potentially humid and the evapotranspiration is under climate control, but the infiltration is under soil control. We call this region 'subhumid.'

The sketch in Figure 11 illustrates this classification mechanically. We will illustrate it numerically through its application to two climate-soil systems, that at Clinton and that at Santa Paula.

Values of the independent climate parameter for these areas, as determined elsewhere [Eagleson, 1978f] by analysis of meteorological data, are given in Table 1. No data on soil or vegetal properties or on water table location were available, thus necessitating the following expedients.

1. The \( h_{0} \) was chosen as 1 mm.
2. The \( k_{n} \) was chosen as unity, thus making \( E[\gamma_{PA}^*] \) independent of \( M \).
3. The water table was assumed to be deep enough to have no influence.
4. The Clinton and Santa Paula soils were chosen as silty loam. Among the four representative soils of Table 2 this gives the best fit when derived yield frequency is compared with observed streamflow frequency [Eagleson, 1978e].

In Table 4 are given the corresponding values of \( m_{PA} \), satisfying classification limits 1, 2, and 3 in Figure 11. From the magnitude of the observed \( m_{PA} \), in relation to these we see that the Clinton system may be classified as subhumid, while the Santa Paula system is arid. The coincidence of the two asymptotes in both cases tells us that for such a porous soil, surface runoff will tend to vanish in very dry years.

SIMILARITY PARAMETERS

Using (24) in (22a) and dividing through by \( m_{PA} \) give the dimensionless average annual water balance equation for soil moisture;

\[ [1 - e^{-G - 2w} \Gamma(\sigma + 1) \sigma^{-1}] = -\frac{E[\gamma_{PA}^*]}{m_{PA} \gamma} J(E, M_{n}, k_{n}) \]

\[ + \frac{m_{PA} K(1)}{m_{PA} \gamma} \frac{s_{0} - Tw}{m_{PA}} \] (62)

which defines the dependent (dimensionless) variable \( s_{0} \) in terms of a set of independent (dimensionless) variables having physical significance and being the similarity parameters for the annual water balance. (In (62) the term to the left of the equal sign is infiltration, the first term to the right is evapotranspiration from soil moisture, and the last two terms are groundwater runoff (the first is groundwater recharge and the last is groundwater loss).)

We identify these parameters as follows:

1. Potential humidity (a climate-soil-vegetation parameter). Potential humidity is

\[ Z = m_{PA}/E[\gamma_{PA}^*] \] (63)

As we have shown earlier, a necessary and sufficient condition for a climate-soil-vegetation system to be fully arid is

\[ Z = m_{PA}/E[\gamma_{PA}^*] < 1 \]

while a necessary (but not sufficient) condition for fully humid behavior is

\[ Z = m_{PA}/E[\gamma_{PA}^*] > 1 \]

2. Pore disconnectedness index c (a soil parameter). As we decrease the interconnectedness of the soil interstitial passages, we expect the soil to be less permeable to fluid flow. In soils with highly interconnected pores, i.e., small \( c \), we expect the permeability to rise rapidly with increasing saturation as additional flow passages are readily brought into play.

3. Gravitational infiltration potential (a climate-soil parameter). Equation (7) can be rewritten

\[ G = \alpha K(1)\left[(1 + s_{0}^{2} - w)/K(1)\right] \] (64)

Remembering [Eagleson, 1978d] that for \( s_{0} = 1, w = 0 \), we can write (64) at saturation as

\[ G(1) = \alpha K(1)/m_{n} \] (65)

which equals the maximum gravitational infiltration rate divided by the average rainfall rate. When

\[ G(1) > 1 \]

the gravitational infiltration is potentially limited climatically, and when

\[ G(1) \leq 1 \]

it is potentially limited by the soil. Whether these are true limits, of course, depends upon the actual value of \( G \) as modified by the factor containing \( s_{0} \), which depends in turn upon the full set of soil, climate, and vegetation parameters. The factor \( G \) is of major importance in dividing the annual precipitation into its infiltration and rainfall excess components. In general, when \( G(1) > 1 \), gravitational effects will dominate the infiltration process and produce large infiltration with little rainfall excess. When \( G(1) \leq 1 \), the rainfall excess component becomes important.

We will thus define the gravitational infiltration potential as

\[ G(1) = \alpha K(1) \] (66)

4. Index of water table influence (a climate-soil parameter). The second bracketed term of (64) represents the relative dynamic importance of the water table presence, since...
w/K(1) is the characteristic capillary rise velocity divided by the characteristic gravitational percolation velocity. We will let
\[ T = \frac{w}{K(1)} \] (67)
be the index of water table influence. For
\[ T = \frac{w}{K(1)} \ll 1 \]
the water table is of negligible importance to the dynamics of the soil moisture exchange process. For
\[ T = \frac{w}{K(1)} \geq O(1) \]
the water table is very important, and we must question the superposition approximation used to incorporate its effects in this analysis.

5. **Capillary infiltration effectiveness** (a climate-soil parameter). From its derivation [Eagleson, 1978b] we see that \((2\sigma)^{1/2}\) is the characteristic capillary infiltration depth divided by the average storm depth. Specializing \(\sigma\) as defined in (8) for the condition \(S_0 = 0\), at which capillary infiltration will be a maximum, we obtain
\[ \sigma = \left[\sigma(0) + 5/3(1 - S_0)\right]^{1/2} \] (68)
in which
\[ \sigma(0) = \frac{5m\Psi K(1)}{6\pi d-md+5/3} \] (69)
From the above definitions we see that when
\[ 2\sigma(0) > 1 \]
the maximum characteristic capillary infiltration exceeds the storm depth. We thus define \(2\sigma(0)\) as the capillary infiltration effectiveness.

Along with \(G(0), 2\sigma(0)\) governs the slope of the arid asymptote of \(I_a\) and \(E_{R*}\), as defined in (45). For \(2\sigma(0) > 1\) the infiltration is high and rainfall excess becomes unimportant. For \(2\sigma(0) < 1\) the rainfall excess becomes important.

6. **Exfiltration effectiveness** (a climate-soil parameter). From its derivation [Eagleson, 1978c], we see that \(2E(1)^{1/2}\) is the characteristic interstorm exfiltration depth divided by the characteristic potential interstorm evaporation depth. Specializing \(E\) as defined in (19) for the condition \(S_0 = 1\), at which exfiltration will have its maximum value, we can write
\[ E = E(1)\sigma^{1/2} \] (70)
where
\[ E(1) = \frac{2m\Psi K(1)}{\pi m \sigma^{3/2}} \] (71)
From the above definitions we see that when
\[ 2E(1) > 1 \]
the maximum characteristic interstorm exfiltration exceeds the characteristic potential interstorm evaporation. We thus define \(2E(1)\) as the exfiltration effectiveness.

7. **Potential transpiration efficiency** (a vegetal parameter). The ratio of potential rates of transpiration and soil surface evaporation defines the vegetal potential transpiration efficiency \(k_v\). That is, \(k_v\) equals the potential rate of transpiration divided by the potential (soil surface) rate of evaporation. This is often called the 'plant coefficient' by agriculturists.

8. **Groundwater recharge potential** (a climate-soil parameter). The groundwater runoff terms of (62) can be written
\[ \frac{E[R_{*a}]}{m_{P_a}} = \frac{m_{K(1)}}{m_{P_a}} \left[\sigma - \left(T/m_r\right)w/K(1)\right] \] (72)
The factor \(m_{K(1)}/m_{P_a}\) represents the potential unit wet season groundwater recharge, since it is the value of the recharge term in (62), when \(S_0 = 1\). It is called the groundwater recharge potential
\[ \Omega = m_rK(1)/m_{P_a} \] (73)
When
\[ \Omega = m_rK(1)/m_{P_a} > 0 \]
the groundwater recharge potential is climate limited. When the ratio is less than one, the potential is soil limited. It must be remembered, however, that this is only a potential, and its realization depends upon the value of the modifying factor \(\sigma^c\). The value of \(\sigma^c\) is dependent upon all the other soil, climate, and vegetation parameters and processes.

9. **Groundwater loss index** (a climate-soil parameter). The second bracketed term of (72) represents the potential fraction of maximum groundwater recharge which is lost to evapotranspiration through capillary rise from the water table. That is, \(T/m_rK(1)\) is equal to the annual volume of capillary rise from the water table divided by the maximum wet season recharge of groundwater. It measures the volumetric importance of water table presence in the annual water balance. We will define
\[ \Lambda = \frac{T_w}{m_rK(1)} \] (74)
as the groundwater loss index.
To the extent that
\[ \Lambda = T_w/m_rK(1) \ll 1 \]
the water table will have a negligible effect on groundwater losses.

We have defined the annual water budget in terms of nine independent dimensionless parameters. One is a soil parameter, one is a vegetal parameter, one is a climate-soil-vegetation parameter, and six are climate-soil-parameters.

For the special case of negligible water table influence the parameter set is reduced to seven, one soil, one vegetal, one climate-soil-vegetation, and four climate-soil.

**SUMMARY AND CONCLUSIONS**

The average annual components of soil moisture movement (infiltration, evapotranspiration, and percolation to groundwater) expressed in terms of physical properties and parameters of the climate, soil, and vegetation are combined in a statement of mass conservation to yield a water balance equation.

A hypothesis of natural selection is proposed that consists of the following propositions.
1. Proposition 1 states that natural vegetal systems operate in an equilibrium state which is unstrained on the average.
2. Proposition 2 states that natural vegetal systems exist at an equilibrium density and root configuration which fully exploits the soil moisture above the maximum root depth.
3. Proposition 3 states that natural vegetal systems develop a growth equilibrium state in which average soil moisture is maximized.
4. Proposition 4 states that equilibrium natural vegetal systems which are water limited evolve toward maximum water utilization.
This hypothesis leads to specification of the equilibrium vegetal canopy density and of the plant coefficient $k_c$ for water-limited natural systems in terms of the system exfiltration effectiveness $E$.

A sensitivity analysis of the water balance equation demonstrates the critical importance of the exfiltration effectiveness in characterizing the behavior of climate-soil-vegetation systems.

An asymptotic analysis of this equation yields rational criteria for classifying climate-soil-vegetation systems during the rainy season.

The dimensionless form of the equation yields the set of nine physically significant dimensionless parameters which define the conditions for water balance similarity among regions of the world.

**Notation**

$c$ pore disconnectedness index.

$d$ diffusivity index.

$E$ exfiltration parameter.

$E_{p_a}$ average annual potential evapotranspiration, centimeters.

$E_{p_s}$ average annual potential soil moisture evapotranspiration, centimeters.

$E_r$ storm surface retention, centimeters.

$E_{sa}$ annual surface retention, centimeters.

$E_{ta}$ annual evapotranspiration, centimeters.

$E_{ta^*}$ annual evapotranspiration from soil moisture, centimeters.

$\bar{e}_p$ long-term time average rate of potential (soil surface) evaporation, centimeters per second.

$\bar{e}_{p^*}$ long-term time average rate of potential evapotranspiration from soil moisture, centimeters per second.

$G$ gravitational infiltration parameter.

$G(0)$ dry soil gravitational infiltration parameter.

$G(1)$ saturated soil gravitational infiltration parameter, which equals gravitational infiltration potential.

$h_0$ surface retention capacity, centimeters.

$l_a$ annual infiltration, centimeters.

$K(1)$ saturated hydraulic conductivity, centimeters per second.

$k_o$ potential transpiration efficiency, which equals plant coefficient.

$k(1)$ saturated intrinsic permeability, square centimeters.

$M$ vegetal canopy density.

$M_a$ equilibrium vegetal canopy density.

$m$ pore size distribution index.

$m_{st}$ mean storm depth, centimeters.

$m_{st0}$ mean storm intensity, seconds per centimeter.

$m_{a}$ average annual precipitation, centimeters.

$m_{b}$ mean time between storms, days.

$m_{c}$ mean storm duration, days.

$m_{n}$ mean number of storms per year.

$m_{r}$ mean length of rainy season, days.

$n$ medium effective porosity, which equals volume of active voids divided by total volume.

$F_p$ annual precipitation, centimeters.

$R_{sa}$ annual groundwater runoff, centimeters.

$R_{ta}$ annual surface runoff, centimeters.

$R_{ta^*}$ annual rainfall excess, centimeters.

$\Delta S_{sa}$ annual change in surface storage, centimeters.

$\Delta S_{ta}$ annual change in soil moisture and groundwater storage, centimeters.

$s_a$ time and spatial average soil moisture concentration in surface boundary layer.

$T$ one year, seconds.

$w$ upward apparent fluid velocity representing capillary rise from the water table, centimeters per second.

$z$ depth to water table, centimeters.

$\alpha$ reciprocal of average rainstorm intensity, equal to $m_{a}^{-1}$, seconds per centimeter.

$\beta$ reciprocal of average time between storms, equal to $m_{b}^{-1}$, days$^{-1}$.

$\delta$ reciprocal of average storm duration equal to $m_{c}^{-1}$, days$^{-1}$.

$\eta$ reciprocal of mean storm depth, equal to $m_{st0}^{-1}$, cm$^{-1}$.

$\kappa$ parameter of gamma distribution of storm depth.

$\Lambda$ groundwater loss index.

$\lambda$ parameter of gamma distribution of storm depths, equal to $\kappa/m_{st0}$, cm$^{-1}$.

$\Xi$ potential humidity.

$\sigma(0)$ dry soil capillary infiltration parameter.

$\tau$ length of rainy season, days.

$\Gamma$ index of water table influence.

$\phi_a$ dimensionless exfiltration diffusivity.

$\phi$ dimensionless infiltration diffusivity.

$\Psi(1)$ saturated soil matrix potential, centimeters (suction).

$\Omega$ groundwater recharge potential.

$E(\cdot)$ expected value of $[\cdot]$.

$I(\cdot)$ evapotranspiration function.

$\Gamma(\cdot)$ gamma function.

$\gamma[a, x]$ incomplete gamma function.

**Acknowledgments.** This work was performed in part during the 1975-1976 academic year while the author was a visiting associate in the Environmental Quality Laboratory at the California Institute of Technology on sabbatical leave from MIT. The sensitivity analysis was performed with the assistance of Pedro Restrepo, research assistant in the Department of Civil Engineering at MIT under NSF grant ENG 76-11236. Publication was made possible by a grant from the Sloan Basic Research Fund of MIT. The author is indebted to Francois M. M. Morel, associate professor and to Harold Hemond, assistant professor, both of the Civil Engineering Department of MIT, for their helpful discussions of the biological aspects of this work.

**References**


(Received August 23, 1977; revised January 6, 1978; accepted February 16, 1978.)